Use a Normal approximation to find the probability of obtaining at least 60 heads when spinning a fair coin 100 times.

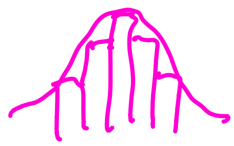
$X \sim B(\overset{n}{100}, \overset{p}{0.5})$ require $P(X \geq 60)$

$E(x) = np = 50$

$Var(x) = npq = 25$

Approx with μ, σ^2

$Y \sim N(50, 5^2)$



continuity correction

$P(X \geq 60) \approx P(Y > 59.5) = 0.0287$

Using Binomial

$P(X \geq 60) = 1 - P(X \leq 59)$

$= 1 - 0.9716$

$= 0.0284$

Jun 10

3 In a men's cycling time trial, the times are modelled by the random variable X minutes which is Normally distributed with mean 63 and standard deviation 5.2.

(i) Find

(A) $\mathrm{P}(X < 65)$,

(B) $\mathrm{P}(60 < X < 65)$. **[6]**

(ii) Find the probability that 5 riders selected at random all record times between 60 and 65 minutes. **[2]**

(iii) A competitor aims to be in the fastest 5% of entrants (i.e. those with the lowest times). Find the maximum time that he can take. **[3]**

It is suggested that holding the time trial on a new course may result in lower times. To investigate this, a random sample of 15 competitors is selected. These 15 competitors do the time trial on the new course. The mean time taken by these riders is 61.7 minutes. You may assume that times are Normally distributed and the standard deviation is still 5.2 minutes. A hypothesis test is carried out to investigate whether times on the new course are lower.

(iv) Write down suitable null and alternative hypotheses for the test. Carry out the test at the 5% significance level. **[8]**

i) $X \sim N(\overset{\mu}{63}, \overset{\sigma^2}{5.2^2})$

A) $P(x < 65) = 0.6497$

B) $P(60 < x < 65) = 0.3677$

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ii) $X \sim B(\overset{n}{5}, \overset{p}{0.3677})$

$P(x = 5) = 0.3677^5 = 0.0067$

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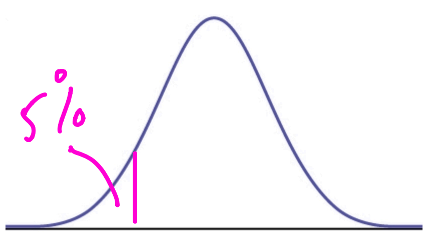
iii)



$X \sim N(63, 5.2)$

$\Phi^{-1}(0.05) = 54.45$ min

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iv) $X \sim N(63, 5.2^2)$

$H_0: \bar{X} = 63$

$H_1: \bar{X} < 63$

$\bar{X} \sim N\left(63, \left(\frac{5.2}{\sqrt{15}}\right)^2\right)$

$P(\bar{X} \leqslant 61.7) = 0.1665 > 5\%$

Accept H_0

There is not sufficient evidence to suggest the mean time is lower on the new course