Chapter 4


Binomial Expansion
4.2 Expanding $(a+b x)^{\wedge} n$

Edexcel A level Mathematics
Pure Mathematics
Year 2

$$
\begin{aligned}
\text { Expanding } & (a+b x)^{n} \\
= & \left(a\left(1+\frac{b}{a} x\right)\right)^{n} \\
= & a^{n}\left(1+\frac{b}{a} x\right)^{n}
\end{aligned}
$$

Expand $(2+3 x)^{-2}$ as far as the term in $x^{3}$ and state the range of values for which the expansion is valid

$$
\begin{aligned}
& (2+3 x)^{-2}=\left(2\left(1+\frac{3 x}{2}\right)\right)^{-2}=2^{-2}\left(1+\frac{3 x}{2}\right)^{-2} \\
\approx & \frac{1}{4}\left[1-2\left(\frac{3 x}{2}\right)+\frac{-x \cdot-3}{1 \cdot x}\left(\frac{3 x}{2}\right)^{2}+\frac{-2 \cdot-3 \cdot-4}{1 \cdot x \cdot 3}\left(\frac{3 x}{2}\right)^{3}\right] \\
= & \frac{1}{4}\left[1-3 x+\frac{27}{4} x^{2}-\frac{27}{2} x^{3}\right] \\
= & \frac{1}{4}-\frac{3}{4} x+\frac{27}{16} x^{2}-\frac{27}{8} x^{3}
\end{aligned}
$$

Valid for $\quad\left|\frac{3 x}{2}\right|<1$

$$
\begin{aligned}
|x| & <\frac{2}{3} \\
-\frac{2}{3} & <x<\frac{2}{3}
\end{aligned}
$$

Expand $\frac{2+x}{\sqrt{4+x}}$ as for as the term in $x^{3}$ and state the range of values for which the expansion is valid

$$
\begin{aligned}
& \quad \frac{2+x}{\sqrt{4\left(1+\frac{x}{4}\right)}}=\frac{2+x}{2 \sqrt{1+\frac{x}{4}}}=\frac{1}{2}(2+x)\left(1+\frac{x}{4}\right)^{-\frac{1}{2}} \\
& \approx\left(1+\frac{x}{2}\right)\left[1+-\frac{1}{2}\left(\frac{x}{4}\right)+\frac{-\frac{1}{2} \cdot-\frac{3}{2}}{1 \cdot 2}\left(\frac{x}{4}\right)^{2}+\frac{\left.-\frac{1}{2} \cdot \frac{-\frac{5}{2} \cdot \frac{-5}{2}}{1 \cdot 2 \cdot 3}\left(\frac{x}{4}\right)^{3}\right]}{=} \begin{array}{l}
\left(1+\frac{x}{2}\right)\left[1-\frac{x}{8}+\frac{3}{128} x^{2}-\frac{5}{1024} x^{3}\right] \\
=\quad 1-\frac{x}{8}+\frac{3}{128} x^{2}-\frac{5}{1024} x^{3} \\
\\
\quad+\frac{x}{2}-\frac{x^{2}}{16}+\frac{3}{256} x^{3} \\
=\quad 1
\end{array} \quad+\frac{3}{8} x-\frac{5}{128} x^{2}+\frac{7}{1024} x^{3}\right.
\end{aligned}
$$

Valid for $\left|\frac{x}{4}\right|<1$

$$
\begin{array}{r}
|x|<4 \\
-4<x<4
\end{array}
$$

