NON-LINEAR SIMULTANEOUS EQUATIONS

Example 1

Solve

$$y = x^2 - 3x + 1 \quad ①$$
$$y = x^2 + x - 7 \quad ②$$

Subst for y in ①

$$x^2 + x - 7 = x^2 - 3x + 1$$
$$\cancel{x^2} + x - 7 - \cancel{x^2} + 3x - 1 = 0$$
$$4x - 8 = 0$$
$$4x = 8$$
$$x = \frac{8}{4}$$
$$x = 2$$

Subst for x in ①

$$y = 2^2 - 3(2) + 1$$
$$y = 4 - 6 + 1$$
$$y = -1$$

Solution $\begin{cases} x = 2 \\ y = -1 \end{cases}$

Example 2

Solve

$$y = 2x^2 - 3x - 5 \quad ①$$
$$y = x^2 - 4x - 3 \quad ②$$

Subst for y in ②

$$2x^2 - 3x - 5 = x^2 - 4x - 3$$
$$2x^2 - 3x - 5 - x^2 + 4x + 3 = 0$$
$$x^2 + x - 2 = 0$$
$$(x + 2)(x - 1) = 0$$
$$\Rightarrow x = -2 \quad \text{or} \quad x = +1$$

Subst for x in ①.

when $x = -2$, $y = (-2)^2 - 4(-2) - 3$
$$y = 4 + 8 - 3 = 9$$

so $\underline{x = -2, y = 9}$

When $x = +1$, $y = 1^2 - 4(1) - 3$
$$y = 1 - 4 - 3 = -6$$

so $\underline{x = 1, y = -6}$

Solutions:

$\begin{cases} x = -2 \\ y = 9 \end{cases}$ $\qquad$ $\begin{cases} x = 1 \\ y = -6 \end{cases}$

Example 3

Leaving answers in surd form solve

$$y = x^2 + 4x + 3 \qquad ①$$

$$y = 2x + 4 \qquad ②$$

Subst for y in ①

$$2x + 4 = x^2 + 4x + 3$$

$$0 = x^2 + 4x + 3 - 2x - 4$$

$$0 = x^2 + 2x - 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm \sqrt{8}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Find y coords, subst for x in ②

$$x = -1 + \sqrt{2}$$

$$\Rightarrow y = 2(-1 + \sqrt{2}) + 4$$

$$y = -2 + 2\sqrt{2} + 4$$

$$y = 2 + 2\sqrt{2}$$

So $\underline{x = -1 + \sqrt{2}, \quad y = 2 + 2\sqrt{2}}$

When $x = -1 - \sqrt{2}$

$$y = 2(-1 - \sqrt{2}) + 4$$

$$y = -2 - 2\sqrt{2} + 4$$

$$y = 2 - 2\sqrt{2}$$

so $\underline{x = -1 - \sqrt{2}, \quad y = 2 - 2\sqrt{2}}$

Solutions

$$\begin{cases} x = -1 + \sqrt{2} \\ y = 2 + 2\sqrt{2} \end{cases}$$

$$\begin{cases} x = -1 - \sqrt{2} \\ y = 2 - 2\sqrt{2} \end{cases}$$

NON-LINEAR SIMULTANEOUS EQUATIONS   TRANSCRIPT

Example 4   Solve

$$x^2 + y^2 = 25 \qquad ①$$

$$y = x + 1 \qquad ②$$

Subst for y in ①

$$x^2 + (x+1)^2 = 25$$

$$x^2 + x^2 + 2x + 1 = 25$$

$$2x^2 + 2x + 1 - 25 = 0$$

$$2x^2 + 2x - 24 = 0$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$\Rightarrow \quad x = -4 \text{ or } x = +3$$

When $x = -4$
$$y = -4 + 1 = -3$$

When $x = 3$
$$y = 3 + 1 = 4$$

Solutions:

$$\begin{cases} x = -4 \\ y = -3 \end{cases} \qquad \begin{cases} x = 3 \\ y = 4 \end{cases}$$

Example 5   Solve

$$y = \frac{8}{x-5} \qquad ①$$

$$y = x + 2 \qquad ②$$

Subst for y in ①

$$x + 2 = \frac{8}{x-5}$$

$$(x+2)(x-5) = 8$$

$$x^2 + 2x - 5x - 10 = 8$$

$$x^2 - 3x - 10 - 8 = 0$$

$$x^2 - 3x - 18 = 0$$

$$(x-6)(x+3) = 0$$

$$\Rightarrow \quad x = 6 \text{ or } x = -3$$

When $x = 6$
$$y = 6 + 2 = 8$$

When $x = -3$
$$y = -3 + 2 = -1$$

Solutions:

$$\begin{cases} x = 6 \\ y = 8 \end{cases} \qquad \begin{cases} x = -3 \\ y = -1 \end{cases}$$