

Homework Review

- 1 Solve the following equations for $0^\circ \leq \theta \leq 360^\circ$.
(i) $2\sin 2\theta = \cos \theta$ (ii) $\tan 2\theta = 4\tan \theta$ (iii) $\cos 2\theta + \sin \theta = 0$
(iv) $\tan \theta \tan 2\theta = 1$ (v) $2\cos 2\theta = 1 + \cos \theta$
- 2 Solve the following equations for $-\pi \leq \theta \leq \pi$.
(i) $\sin 2\theta = 2\sin \theta$ (ii) $\tan 2\theta = 2\tan \theta$ (iii) $\cos 2\theta - \cos \theta = 0$
(iv) $1 + \cos 2\theta = 2\sin^2 \theta$ (v) $\sin 4\theta = \cos 2\theta$
(Hint: Write the expression in part (v) as an equation in 2θ .)
- 3 By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, express $\sin 3\theta$ in terms of $\sin \theta$.
Hence solve the equation $\sin 3\theta = \sin \theta$ for $0 \leq \theta \leq 2\pi$.
- 4 Solve $\cos 3\theta = 1 - 3\cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$.
- 5 Simplify $\frac{1 + \cos 2\theta}{\sin 2\theta}$.
- 6 Express $\tan 3\theta$ in terms of $\tan \theta$.
- 7 Show that $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$.

i) $2\sin 2\theta = \cos \theta$

$$4\sin \theta \cos \theta = \cos \theta$$

$$4\sin \theta \cos \theta - \cos \theta = 0$$

$$\cos \theta(4\sin \theta - 1) = 0$$

Either $\cos \theta = 0$

$$\theta = 90^\circ$$

$$\theta = 270^\circ$$

or $4\sin \theta - 1 = 0$

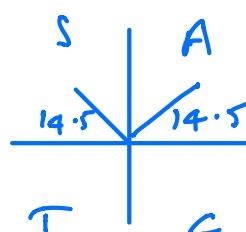
$$4\sin \theta = 1$$

$$\sin \theta = \frac{1}{4}$$

$$\theta = \sin^{-1} \frac{1}{4}$$

$$\theta = 14.5^\circ$$

$$\theta = 165.5^\circ$$



$$\text{Iii) } \tan 2\theta = 4 \tan \theta$$

$$\frac{2 \tan \theta}{1 - \tan^2 \theta} = 4 \tan \theta$$

$$2 \tan \theta = 4 \tan \theta (1 - \tan^2 \theta)$$

$$2 \tan \theta = 4 \tan \theta - 4 \tan^3 \theta$$

$$4 \tan^3 \theta - 2 \tan \theta = 0$$

$$2 \tan \theta (2 \tan^2 \theta - 1) = 0$$

$$\text{Either } \tan \theta = 0$$

$$\theta = 0^\circ$$

$$\theta = 180^\circ$$

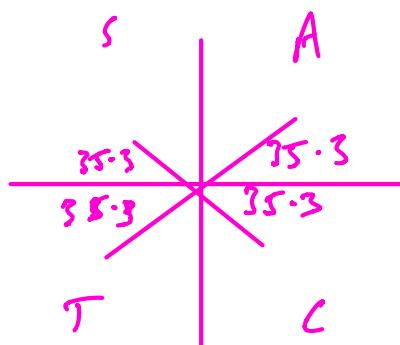
$$\theta = 360^\circ$$

$$\text{or } 2 \tan^2 \theta - 1 = 0$$

$$2 \tan^2 \theta = 1$$

$$\tan^2 \theta = \frac{1}{2}$$

$$\tan \theta = \pm \sqrt{\frac{1}{2}}$$



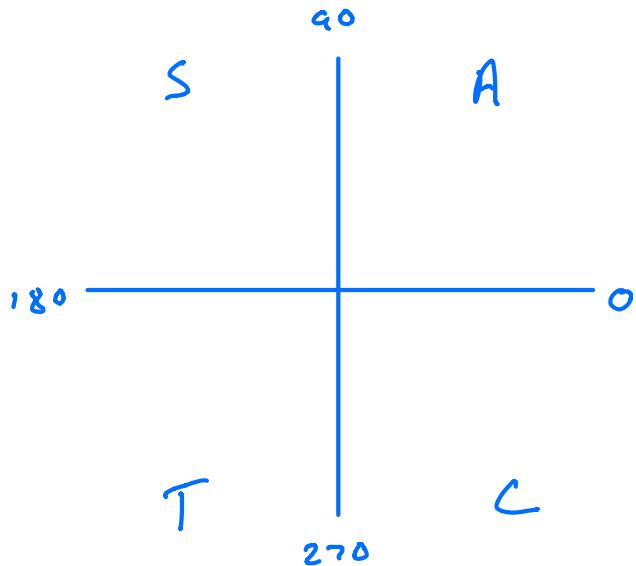
$$\theta = 35.3^\circ$$

$$\theta = 144.7^\circ$$

$$\theta = 215.3^\circ$$

$$\theta = 324.7^\circ$$

CAST DIAGRAM

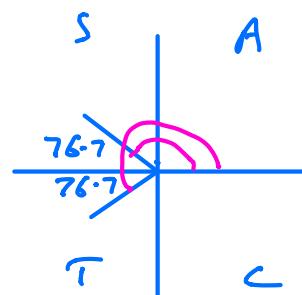


$$\text{Solve } \cos \theta = -0.23$$

$$\cos^{-1} 0.23 = 76.7^\circ$$

$$\theta = 103.3^\circ$$

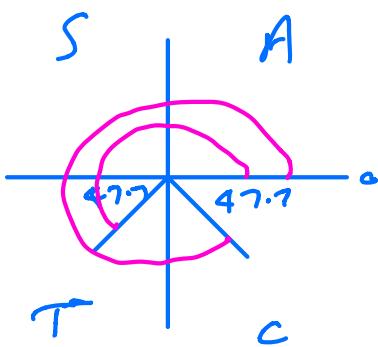
$$\theta = 256.7^\circ$$



Examples Solve for $0^\circ \leq \theta \leq 360^\circ$

$$\sin 2\theta = -0.74$$

$$\sin^{-1} 0.74 = 47.73^\circ$$



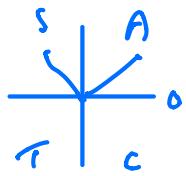
$$\begin{aligned} 2\theta &= 227.73 & \theta &= 113.9^\circ \\ 2\theta &= 312.27 & \theta &= 156.1^\circ \\ 2\theta &= 587.73 & \theta &= 293.9^\circ \\ 2\theta &= 672.27 & \theta &= 336.1^\circ \end{aligned}$$

Exercise

for $0 \leq \theta \leq 360$

Solve

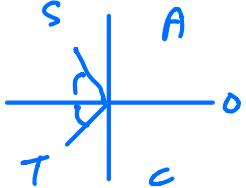
1) $\sin \theta = 0.83$



$$\sin^{-1} 0.83 = 56.1^\circ$$

$$\theta = 56.1^\circ$$
$$\theta = 123.9^\circ$$

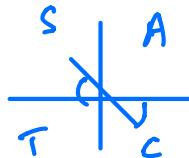
2) $\cos \theta = -0.123$



$$\cos^{-1}(-0.123) = 82.8^\circ$$

$$\theta = 97.1^\circ$$
$$\theta = 262.9^\circ$$

3) $\tan \theta = -1.4$

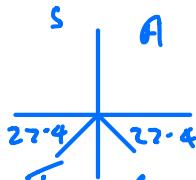


$$\theta = 125.5^\circ$$
$$\theta = 305.5^\circ$$

$$\tan^{-1} 1.4 = 54.5^\circ$$

For $-180 \leq \theta \leq 180$

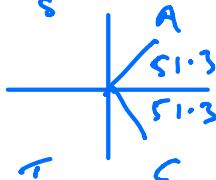
4) $\sin \theta = -0.46$



$$\sin^{-1} -0.46 = -27.4^\circ$$

$$\theta = -27.4^\circ$$
$$\theta = -152.6^\circ$$

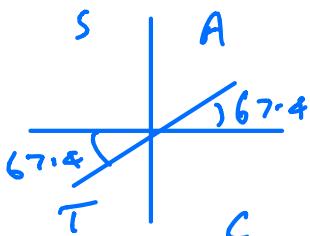
5) $\cos \theta = 0.625$



$$\cos^{-1} 0.625 = 51.3^\circ$$

$$\theta = 51.3^\circ$$
$$\theta = -51.3^\circ$$

6) $\tan \theta = 2.4$



$$\tan^{-1} 2.4 = 67.4^\circ$$

$$\theta = 67.4^\circ$$
$$\theta = -112.6^\circ$$

2 Solve the following equations for $-\pi \leq \theta \leq \pi$.

(i) $\sin 2\theta = 2 \sin \theta$

(ii) $\tan 2\theta = 2 \tan \theta$

(iii) $\cos 2\theta - \cos \theta = 0$

(iv) $1 + \cos 2\theta = 2 \sin^2 \theta$

(v) $\sin 4\theta = \cos 2\theta$

2 i) $\sin 2\theta = 2 \sin \theta$

$$2 \sin \theta \cos \theta = 2 \sin \theta$$

$$2 \sin \theta \cos \theta - 2 \sin \theta = 0$$

$$2 \sin \theta (\cos \theta - 1) = 0$$

Either $\sin \theta = 0$

or $\cos \theta - 1 = 0$

$$\cos \theta = 1$$

$$\theta = -\pi$$

$$\theta = 0$$

$$\theta = \pi$$

$$\theta = 0$$

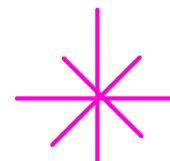
2 iv) $1 + \cos 2\theta = 2 \sin^2 \theta$

$$1 + 1 - 2 \sin^2 \theta = 2 \sin^2 \theta$$

$$2 = 4 \sin^2 \theta$$

$$\frac{1}{2} = \sin^2 \theta$$

$$\pm \frac{1}{\sqrt{2}} = \sin \theta$$



$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, -\frac{\pi}{4}, -\frac{3\pi}{4}$$

Alternative Method

$$1 + \cos 2\theta = 2 \sin^2 \theta$$

$$1 + \cos 2\theta = 1 - \cos 2\theta$$

$$2 \cos 2\theta = 1 - 1$$

$$\cos 2\theta = 0$$

$$\text{If } -\pi \leq \theta \leq \pi$$

$$\Rightarrow 2\theta = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$$

$$-2\pi \leq 2\theta \leq 2\pi$$

$$\theta = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

Alternative Method

$$1 + \cos 2\theta = 2 \sin^2 \theta$$

$$1 + 2 \cos^2 \theta - 1 = 2 \sin^2 \theta$$

$$2 \cos^2 \theta = 2 \sin^2 \theta$$

$$1 = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\tan \theta = \pm \sqrt{1} = \pm 1$$

$$\theta = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

4 Solve $\cos 3\theta = 1 - 3 \cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

$$\cos(2\theta + \theta) = 1 - 3 \cos \theta$$

$$\cos 2\theta \cos \alpha - \sin 2\theta \sin \alpha = 1 - 3 \cos \alpha$$

$$\cos 2\alpha \cos \alpha - 2 \sin^2 \alpha \cos \alpha = 1 - 3 \cos \alpha$$

$$(1 - 2 \sin^2 \alpha) \cos \alpha - 2 \sin^2 \alpha \cos \alpha = 1 - 3 \cos \alpha$$

$$\cos \alpha - 4 \sin^2 \alpha \cos \alpha = 1 - 3 \cos \alpha$$

$$\cos \alpha - 4(1 - \cos^2 \alpha) \cos \alpha = 1 - 3 \cos \alpha$$

$$\cos \alpha - 4 \cos \alpha + 4 \cos^3 \alpha = 1 - 3 \cos \alpha$$

$$\cancel{-3 \cos \alpha} + 4 \cos^3 \alpha = \cancel{1 - 3 \cos \alpha}$$

$$\cos^3 \alpha = \frac{1}{4}$$

$$\cos \alpha = \sqrt[3]{\frac{1}{4}}$$

$$\alpha = 50.95^\circ$$

$$\alpha = 309.05^\circ$$