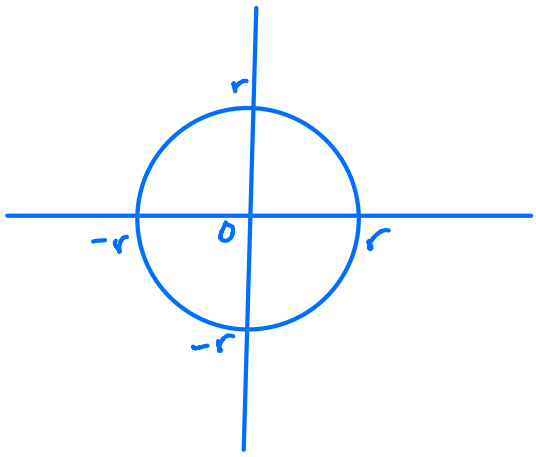
Equation of a Circle

$$x^2 + y^2 = r^2$$

$(0,0)$ is centre $\qquad r$ = radius



Non-Linear Simultaneous Equations

Ex 1

Solve
$$\left.\begin{aligned} x^2 + y^2 &= 25 \\ y &= x + 1 \end{aligned}\right\} \quad \begin{aligned} &① \\ &② \end{aligned}$$

Only use the substitution method

Sub for in ①

$$x^2 + (x+1)^2 = 25$$

$$x^2 + x^2 + 2x + 1 = 25$$

$$2x^2 + 2x + 1 - 25 = 0$$

$$2x^2 + 2x - 24 = 0$$

$$x^2 + x - 12 = 0$$

$$(x-3)(x+4) = 0$$

Either $x - 3 = 0$ or $x + 4 = 0$

$x = 3$ $\quad$ $x = -4$

$y = 3 + 1 = 4$ $\quad$ $y = -4 + 1 = -3$

Solution

$x = 3$ $\quad$ $x = -4$

$y = 4$ $\quad$ $y = -3$

Ex 2

$$y = x^2 - 3x + 5 \quad (1)$$

$$y = 3x + 12 \quad (2)$$

Sub for y in (1)

$$3x + 12 = x^2 - 3x + 5$$

$$0 = x^2 - 3x + 5 - 3x - 12$$

$$0 = x^2 - 6x - 7$$

$$0 = (x + 1)(x - 7)$$

Either $x + 1 = 0$ or $x - 7 = 0$

$x = -1$ $\quad$ $x = 7$

$y = 3(-1) + 12$ $\quad$ $y = 3(7) + 12$

$y = 9$ $\quad$ $y = 33$

$\begin{cases} x = -1 \\ y = 9 \end{cases}$ $\quad$ $\begin{cases} x = 7 \\ y = 33 \end{cases}$

Ex 3

$$y = x^2 + 2x + 8 \quad (1)$$

$$y = 3x + 10 \quad (2)$$

Sub for y in (1)

$$3x + 10 = x^2 + 2x + 8$$

$$0 = x^2 + 2x + 8 - 3x - 10$$

$$0 = x^2 - x - 2$$

$$0 = (x+1)(x-2)$$

Either $x + 1 = 0$ or $x - 2 = 0$

$x = -1$ $\quad$ $x = 2$

$y = 3(-1) + 10$ $\quad$ $y = 3(2) + 10$

$y = 7$ $\quad$ $y = 16$

$$\begin{cases} x = -1 \\ y = 7 \end{cases} \qquad \begin{cases} x = 2 \\ y = 16 \end{cases}$$

Ex 4

$$y = 2x^2 - 3x + 7 \quad (1)$$

$$y = 4x + 5 \quad (2)$$

Sub for y in (1)

$$4x + 5 = 2x^2 - 3x + 7$$

$$0 = 2x^2 - 3x + 7 - 4x - 5$$

$$0 = 2x^2 - 7x + 2$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 2$
$b = -7$
$c = 2$

$$x = \frac{+7 \pm \sqrt{49-16}}{4}$$

$$x = \frac{+7 \pm \sqrt{33}}{4}$$

$$x = \frac{+7+\sqrt{33}}{4} \quad \text{or} \quad x = \frac{+7-\sqrt{33}}{4}$$

$x = 3.186$ $\qquad$ $x = 0.314$

$y = 4(3.186) + 5$ $\qquad$ $y = 4(0.314) + 5$

$y = 17.7$ $\qquad$ $y = 6.26$

$$\begin{cases} x = 3.2 \\ y = 17.7 \end{cases} \qquad \begin{cases} x = 0.3 \\ y = 6.3 \end{cases}$$

Exercise

$$\begin{cases} x^2 + y^2 = 100 & (1) \\ y = 2x + 3 & (2) \end{cases}$$

Sub for y in (1)

$$x^2 + (2x+3)^2 = 100$$

$$x^2 + 4x^2 + 12x + 9 = 100$$

$$5x^2 + 12x + 9 - 100 = 0$$

$$5x^2 + 12x - 91 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \times 5 \times -91}}{2 \times 5}$$

$$x = \frac{-12 \pm \sqrt{1964}}{10}$$

$$x = \frac{-12 + \sqrt{1964}}{10} \quad \text{or} \quad x = \frac{-12 - \sqrt{1964}}{10}$$

$$x = 3.23 \qquad x = -5.63$$

$$y = 2(3.23) + 3 \qquad y = 2(-5.63) + 3$$

$$y = 9.46 \qquad y = -8.26$$