

## Completing the Square

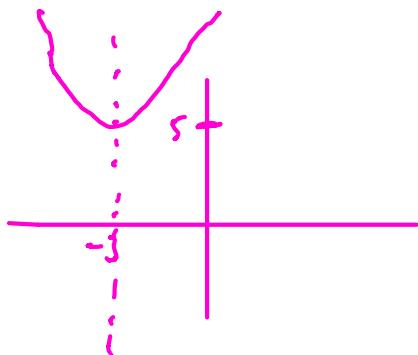
$$\begin{aligned} \text{Consider } (x+a)^2 &= (x+a)(x+a) \\ &= x^2 + ax + ax + a^2 \\ &= x^2 + 2ax + a^2 \end{aligned}$$

Completing the square makes use of this identity

Suppose  $y = x^2 + 6x + 14$

$$y = (x+3)^2 + 14 - 9$$

$$y = (x+3)^2 + 5$$



Sketch graph shows a line of symmetry at  $x = -3$  and a minimum point at  $(-3, 5)$

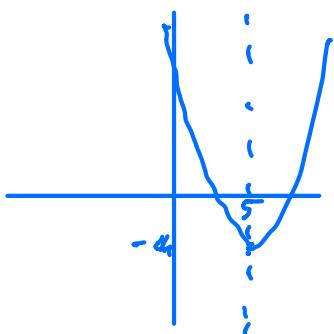
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Ex2  $y = x^2 - 10x + 21$

$$y = (x-5)^2 + 21 - 25$$

$$y = (x-5)^2 - 4$$

$$\text{Min point} = (5, -4)$$



E\*3

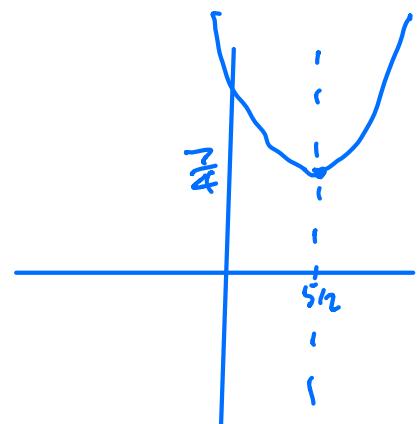
$$y = x^2 - 5x + 8$$

$$y = (x - \frac{5}{2})^2 + 8 - \frac{25}{4}$$

$$y = (x - \frac{5}{2})^2 + \frac{32}{4} - \frac{25}{4}$$

$$y = (x - \frac{5}{2})^2 + \frac{7}{4}$$

$$\text{Min point } (\frac{5}{2}, \frac{7}{4})$$



Exercise

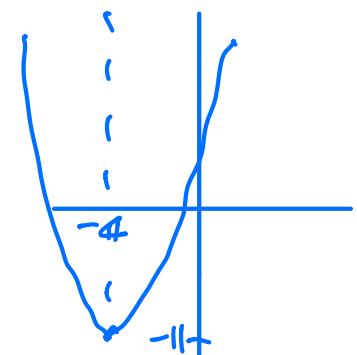
1)

$$y = x^2 + 8x + 5$$

$$y = (x + 4)^2 + 5 - 16$$

$$y = (x + 4)^2 - 11$$

$$\text{Min point } (-4, -11)$$



Line of symmetry  $x = -4$

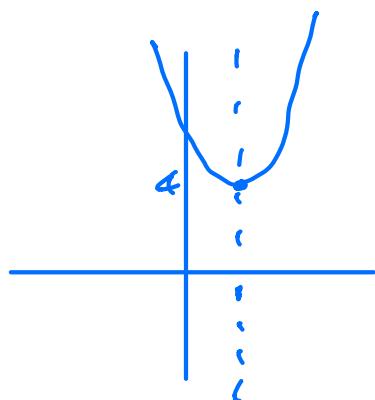
2)

$$y = x^2 - 2x + 5$$

$$y = (x - 1)^2 + 5 - 1$$

$$y = (x - 1)^2 + 4$$

$$\text{Min point } (1, 4)$$



Line of symmetry  $x = 1$

# Solving Quadratic Equations by Completing the Square

Ex1

$$x^2 + 2x - 8 = 0$$

$$(x+1)^2 - 8 - 1 = 0$$

$$(x+1)^2 - 9 = 0$$

$$(x+1)^2 = 9$$

$$x+1 = \pm 3$$

$$x = \pm 3 - 1$$

$$x = +3 - 1 \quad \text{or} \quad x = -3 - 1$$

$$\underline{x = 2}$$

$$\underline{x = -4}$$

Ex2

Solve

$$x^2 - 5x - 6 = 0$$

$$(x - \frac{5}{2})^2 - 6 - \frac{25}{4} = 0$$

$$(x - \frac{5}{2})^2 - \frac{24}{4} - \frac{25}{4} = 0$$

$$(x - \frac{5}{2})^2 - \frac{49}{4} = 0$$

$$(x - \frac{5}{2})^2 = \frac{49}{4}$$

Take Sq roots

$$x - \frac{5}{2} = \pm \frac{7}{2}$$

$$x = \pm \frac{7}{2} + \frac{5}{2}$$

$$x = +\frac{7}{2} + \frac{5}{2} \quad \text{or} \quad x = -\frac{7}{2} + \frac{5}{2}$$

$$x = 6$$

$$x = -1$$

## Exercise

1)

$$\begin{aligned}x^2 - 6x + 8 &= 0 \\(x - 3)^2 + 8 - 9 &= 0 \\(x - 3)^2 - 1 &= 0 \\(x - 3)^2 &= 1 \\x - 3 &= \pm 1 \\x &= \pm 1 + 3 \\x &= +1 + 3 \quad \text{or} \quad x = -1 + 3 \\x &= 4 \qquad \qquad \qquad x = 2\end{aligned}$$

2)

$$\begin{aligned}x^2 + 4x - 5 &= 0 \\(x + 2)^2 - 5 - 4 &= 0 \\(x + 2)^2 - 9 &= 0 \\(x + 2)^2 &= 9 \\x + 2 &= \pm 3 \\x &= \pm 3 - 2 \\x &= +3 - 2 \quad \text{or} \quad x = -3 - 2 \\x &= 1 \qquad \qquad \qquad x = -5\end{aligned}$$

3)

$$\begin{aligned}x^2 - 7x - 8 &= 0 \\(x - \frac{7}{2})^2 - 8 - \frac{49}{4} &= 0 \\(x - \frac{7}{2})^2 - \frac{32}{4} - \frac{49}{4} &= 0\end{aligned}$$

$$(x - \frac{7}{2})^2 - \frac{81}{4} = 0$$

$$(x - \frac{7}{2})^2 = \frac{81}{4}$$

$$x - \frac{7}{2} = \pm \frac{9}{2}$$

$$x = \frac{9}{2} + \frac{7}{2} \quad \text{or} \quad x = -\frac{9}{2} + \frac{7}{2}$$

$$x = \frac{16}{2}$$

$$x = -\frac{2}{2}$$

$$\underline{x = 8}$$

$$\underline{x = -1}$$