Q7.

Prove that

$(2n + 3)^2 - (2n - 3)^2$ is a multiple of 8

for all positive integer values of n.

$= (4n^2 + 12n + 9) - (4n^2 - 12n + 9)$

$= \cancel{4n^2} + 12n + \cancel{9} - \cancel{4n^2} + 12n - \cancel{9}$

$= 24n$

$= 8(3n)$ 8 is a factor

so expression is a multiple of 8

Q11.

Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

$= (4n^2 + 4n + 1) - (2n + 1)$

$= 4n^2 + 4n + 1 - 2n - 1$

$= 4n^2 + 2n$

$= 2(2n^2 + n)$ 2 is a factor

$\therefore$ expression is even

Q13.

Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12

Let 3 consecutive odd numbers be

$2n+1$, $2n+3$, $2n+5$

$$(2n+1)^2 + (2n+3)^2 + (2n+5)^2$$

$$= 4n^2 + 4n + 1 + 4n^2 + 12n + 9 + 4n^2 + 20n + 25$$

$$= 12n^2 + 36n + 35$$

$$= 12n^2 + 36n + 24 + 11$$

$$= 12(n^2 + 3n + 2) + 11$$

This is 11 more than a multiple of 12

Q14.

Prove algebraically that the difference between any two different odd numbers is an even number.

Let $2m+1$, $2n+1$ be any odd integers (where m, n are integers)

$$(2m+1) - (2n+1)$$

$$= 2m + 1 - 2n - 1$$

$$= 2m - 2n$$

$$= 2(m-n)$$

2 is a factor

$\therefore$ number is even

Q9.

The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

Let n, $n+1$ be consecutive integers

$$(n+1) + n(n+1)$$
$$= (n+1)(1+n) \qquad = (n+1)^2$$ a square number

or $$(n+1) + n(n+1)$$
$$= n+1 + n^2 + 1$$
$$= n^2 + 2n + 1$$
$$= (n+1)(n+1) \qquad = (n+1)^2$$ a square number