Calculus Revision

$$\frac{d}{dx} ax^n = nax^{n-1}$$

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$$

for all n except $n=-1$

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1. Find $\int (3x^2 + 4x^5 - 7)\,dx$.

(4)

$$= \frac{3x^3}{3} + \frac{4x^6}{6} - 7x + c$$

$$= x^3 + \frac{2x^6}{3} - 7x + c$$

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5. (a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \frac{2\sqrt{x}+3}{x}$, $\quad x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

a) $$\frac{2\sqrt{x}+3}{x} = \frac{2}{\sqrt{x}} + \frac{3}{x} = 2x^{-\frac{1}{2}} + 3x^{-1}$$

b) $$y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}$$

$$\frac{dy}{dx} = 5 + (-\tfrac{1}{2})2x^{-3/2} + (-1)3x^{-2}$$

$$\frac{dy}{dx} = 5 - x^{-3/2} - 3x^{-2}$$

9. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $f(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

a)
$$f'(x) = 4x - 6x^{\frac{1}{2}} + 8x^{-2}$$

$$f(x) = \frac{4x^2}{2} - \frac{6x^{3/2}}{3/2} + \frac{8x^{-1}}{-1} + c$$

$$f(x) = 2x^2 - 4x^{3/2} - 8x^{-1} + c$$

sus (4, 1)
$$1 = 2(4)^2 - 4(4)^{3/2} - 8(4)^{-1} + c$$

$$1 = 32 - 32 - 2 + c$$

$$1 + 2 = c$$

$$3 = c$$

$$f(x) = 2x^2 - 4x^{3/2} - 8x^{-1} + 3$$

b) When $x = 4$, $f'(x) = 4(4) - 6(4)^{\frac{1}{2}} + 8(4)^{-2}$

$$= 16 - 12 + \frac{8}{16}$$

$$= \frac{9}{2}$$

Gradient of normal $= -\frac{2}{9}$

$$y - y_1 = m(x - x_1)$$

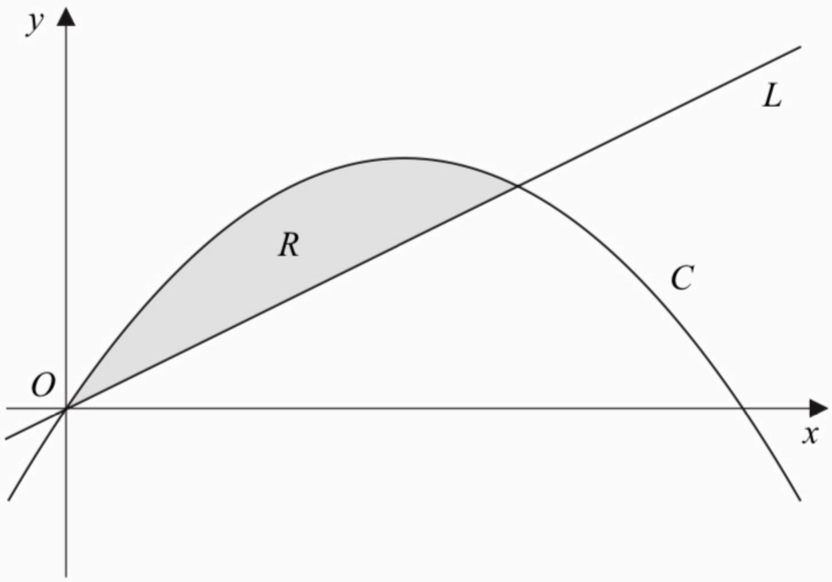
$$y - 1 = -\frac{2}{9}(x - 4)$$

$$9y - 9 = -2x + 8$$

Normal $\underline{2x + 9y - 17 = 0}$

7.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

a) $y = 6x - x^2 = x(6 - x)$

On x-axis $y=0$ $\quad 0 = x(6-x)$

$\Rightarrow x=0$ or $6-x=0$

$x=6$

b) $\begin{cases} y = 2x \\ y = 6x - x^2 \end{cases}$

Sub for y $\quad 2x = 6x - x^2$

$x^2 + 2x - 6x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$\Rightarrow x = 0$ or $x = 4$

$y = 2\times 0$ $\quad y = 2\times 4$

$y = 0$ $\quad y = 8$

Points of intersection $(0,0)$ $\quad (4,8)$

c) $R = \int_0^4 C - L \, dx$

$R = \int_0^4 ((6x - x^2) - (2x)) \, dx$

$R = \int_0^4 (4x - x^2) \, dx$

$R = \left[\frac{4x^2}{2} - \frac{x^3}{3} \right]_0^4$

$R = \left(\frac{4(4)^2}{2} - \frac{4^3}{3} \right) - (0 - 0)$

$R = \frac{64}{2} - \frac{64}{3} = 10\frac{2}{3}$ units^2

9.

Figure 4

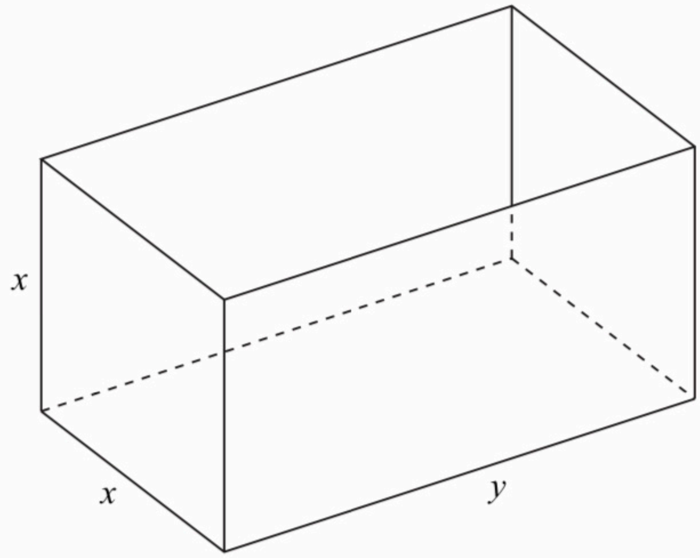


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is $100\,\text{m}^3$.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

a) $A = x^2 + x^2 + xy + xy + xy$

$A = 2x^2 + 3xy$

$V = x^2 y = 100 \text{ m}^3$

$$\Rightarrow \quad y = \frac{100}{x^2}$$

$$\therefore \quad A = 2x^2 + 3x \times \frac{100}{x^2}$$

$$A = 2x^2 + \frac{300}{x}$$

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b) $$A = 2x^2 + 300x^{-1}$$

$$\frac{dA}{dx} = 4x - 300x^{-2}$$

$$\frac{dA}{dx} = 4x - \frac{300}{x^2}$$

At st. pt. $\frac{dA}{dx} = 0 \quad \Rightarrow \quad 4x - \frac{300}{x^2} = 0$

$$4x^3 - 300 = 0$$

$$4x^3 = 300$$

$$x^3 = 75$$

$$x = \sqrt[3]{75}$$

At st. pt. $\quad x = 4.217 \text{ m}$

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c) $$\frac{d^2A}{dx^2} = 4 + 600x^{-3}$$

$$= 4 + \frac{600}{x^3}$$

when $x = 4.217$ $\frac{d^2A}{dx^2} = 4 + \frac{600}{4.217^3} > 0$

$\therefore$ a minimum

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d) Min Area $= 2 \times 4.217^2 + \frac{300}{4.217}$

$= 106.7 \text{ m}^2$

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4. $f(x) = 3x + x^3, \qquad x > 0.$

(a) Differentiate to find $f'(x)$.

(2)

Given that $f'(x) = 15$,

(b) find the value of x.

(3)

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a) $f'(x) = 3 + 3x^2$

b) $15 = 3 + 3x^2$

$12 = 3x^2$

$4 = x^2$

$\underline{x = 2}$ since $x > 0$

9. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k,

(4)

(c) the value of the y-coordinate of A.

(2)

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a) $$\frac{dy}{dx} = 3kx^2 - 2x + 1$$

b) If $2y - 7x + 1 = 0$

$$2y = 7x - 1$$

$$y = \frac{7}{2}x - \frac{1}{2} \qquad \text{gradient} = \frac{7}{2}$$

$\therefore$ gradient of curve at A $= \frac{7}{2}$

$$\Rightarrow 3k\left(-\tfrac{1}{2}\right)^2 - 2\left(-\tfrac{1}{2}\right) + 1 = \frac{7}{2}$$

$$\frac{3k}{4} + 1 + 1 = \frac{7}{2}$$

$$\frac{3k}{4} = \frac{3}{2}$$

$$3k = 6 \qquad \underline{k = 2}$$

c) $y = 2x^3 - x^2 + x - 5$

when $x = -\frac{1}{2}$ $\quad y = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 5$

$y = -\frac{1}{4} - \frac{1}{4} - \frac{1}{2} - 5$

At A $\quad \underline{y = -6}$