

Interpreting Systems of equations

$$A \quad x + y + z = 9$$

$$B \quad x \quad + z = 6$$

$$C \quad 5x + 2y + 5z = 36$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 5 & 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 36 \end{pmatrix}$$

$$\text{Let } M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 5 & 2 & 5 \end{pmatrix}$$

$\det M = 0$ so no inverse

no unique solution

Interpretation

$$x + y + z = 9 \quad ①$$

$$x \quad + z = 6 \quad ②$$

$$5x + 2y + 5z = 36 \quad ③$$

$$\textcircled{3} - 2\textcircled{1} \quad 3x + 3z = 18$$

$$x + z = 6$$

$$\Rightarrow z = 6 - x$$

Sub in \textcircled{1}

$$x + y + 6 - x = 9$$

$$y = 9 - 6$$

$$y = 3$$

Infinite amount of solutions

$$x = \lambda \quad y = 3, \quad z = 6 - \lambda \quad \text{for any } \lambda$$

A sheaf of planes

In Questions 1–10 decide whether the equations are consistent or inconsistent. If they are consistent, solve them, in terms of a parameter if necessary. In each question also describe the configuration of the corresponding lines or planes.

1 $5x + 3y = 31$
 $4x + 2y = 25$

2 $3x + 9y = 12$
 $2x + 6y = 15$

3 $6x + 3y = 12$
 $2x + y = 4$

4 $6x - 3y = 11$
 $y = 2x - 4$

5 $x + y + z = 4$
 $2x + 3y - 4z = 3$
 $5x + 8y - 13z = 8$

6 $2x - y = 1$
 $3x + 2z = 13$
 $3y + 4z = 23$

7 $x + 2y + 4z = 7$
 $3x + 2y + 5z = 2$
 $4x + y + 2z = 14$

8 $3x + 2y + z = 2$
 $5x + 3y - 4z = 1$
 $x + y + 4z = 5$

9 $2x + y - z = 5$
 $8x + 4y - 4z = 20$
 $-2x - y + z = -5$

10 $5x + 3y - 2z = 6$
 $6x + 2y + 3z = 1$
 $7x + y + 8z = 12$

$$5) \underline{M} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & -4 \\ 5 & 8 & -13 \end{pmatrix}$$

$$\underline{M} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{M}^{-1} \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$$

$\det \underline{M} = 0 \therefore$ no unique solution

$$x + y + z = 4 \quad (1)$$

$$2x + 3y - 4z = 3 \quad (2)$$

$$5x + 8y - 13z = 8 \quad (3)$$

$$(3) - 5(1) \qquad 3y - 18z = -12 \quad (4)$$

$$(2) - 2(1) \qquad y - 6z = -5 \quad (5)$$

$$(4) \div 3 \qquad y - 6z = 4$$

inconsistent

Planes form a triangular prism
since none are parallel

$$6 \quad 2x - y = 1$$

$$3x + 2z = 13$$

$$3y + 4z = 23$$

$$\text{Let } \underline{M} = \begin{pmatrix} 2 & -1 & 0 \\ 3 & 0 & 2 \\ 0 & 3 & 4 \end{pmatrix}$$

$$\det \underline{M} = 0$$

no unique solution

$$2x - y = 1 \quad (1)$$

$$3x + 2z = 13 \quad (2)$$

$$3y + 4z = 23 \quad (3)$$

$$(3) - 2(2) \quad -6x + 3y = -3 \quad (4)$$

$$(1) \times 3 \quad -6x + 3y = -3 \quad (5)$$

(4) and (5) are the same

From (1) $\underline{y = 2x - 1}$

Sus for y in (3)

$$3(2x-1) + 4z = 23$$

$$6x - 3 + 4z = 23$$

$$4z = 26 - 6x$$

$$\underline{z = 6.5 - 1.5x}$$

Let $x = \lambda$, $y = 2\lambda - 1$, $z = 6.5 - 1.5\lambda$
for any λ

$$7 \quad \begin{aligned} x + 2y + 4z &= 7 \\ 3x + 2y + 5z &= 2 \\ 4x + y + 2z &= 14 \end{aligned}$$

$$\text{Let } \underline{M} = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 2 & 5 \\ 4 & 1 & 2 \end{pmatrix}$$

$$\det \underline{M} = 7$$

unique solution

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{M}^{-1} \begin{pmatrix} 7 \\ 2 \\ 14 \end{pmatrix} = \begin{pmatrix} 3 \\ 24 \\ -11 \end{pmatrix}$$

$$x = 3, \quad y = 24, \quad z = -11$$
