SURDS

TRANSCRIPT

Consider

$$(\sqrt{a}\times\sqrt{b})\times(\sqrt{a}\times\sqrt{b})$$

$$= \sqrt{a}\times\sqrt{b}\times\sqrt{a}\times\sqrt{b}$$

$$= \sqrt{a}\times\sqrt{a}\times\sqrt{b}\times\sqrt{b}$$

$$= a\times b = ab$$

So $(\sqrt{a}\times\sqrt{b})$ multiplied by itself is equal to ab

It is therefore the square root of ab

$$\sqrt{a}\times\sqrt{b} = \sqrt{ab}$$

Now consider

$$\frac{\sqrt{a}}{\sqrt{b}}\times\frac{\sqrt{a}}{\sqrt{b}}$$

$$= \frac{\sqrt{a}\times\sqrt{a}}{\sqrt{b}\times\sqrt{b}}$$

$$= \frac{a}{b}$$

So $\frac{\sqrt{a}}{\sqrt{b}}$ is the square root of $\frac{a}{b}$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

CAUTION!

$$\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$$

(does not equal)

Example

$$\sqrt{9}+\sqrt{4} \neq \sqrt{9+4}$$

$$3+2 \neq \sqrt{13}$$

$$5 \neq 3\cdot 6$$

Also

$$\sqrt{a}-\sqrt{b} \neq \sqrt{a-b}$$

Example

$$\sqrt{9}-\sqrt{4} \neq \sqrt{9-4}$$

$$3-2 \neq \sqrt{5}$$

$$1 \neq 2.2$$

These examples are called counter-examples which are used to disprove rules that are not valid.

So the two rules that do work are $\sqrt{a}\times\sqrt{b} = \sqrt{ab}$

and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

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Three main tasks:

Task 1. Simplify expressions such as:

$\sqrt{8} + \sqrt{18} + \sqrt{50}$

We check to see if the numbers under the sq root signs contain perfect squares as factors (eg 4, 9, 16, 25, 36, 49, 64, 81, 100)

$\sqrt{8} + \sqrt{18} + \sqrt{50}$

$= \sqrt{4\times2} + \sqrt{9\times2} + \sqrt{25\times2}$

$= \sqrt{4}\times\sqrt{2} + \sqrt{9}\times\sqrt{2} + \sqrt{25}\times\sqrt{2}$

$= 2\sqrt{2} + 3\sqrt{2} + 5\sqrt{2}$

$= 10\sqrt{2}$

2nd example

$\sqrt{20} + 2\sqrt{45} + 3\sqrt{5}$

$= \sqrt{4\times5} + 2\sqrt{9\times5} + 3\sqrt{5}$

$= 2\sqrt{5} + 2\times3\sqrt{5} + 3\sqrt{5}$

$= 2\sqrt{5} + 6\sqrt{5} + 3\sqrt{5}$

$= 11\sqrt{5}$

Task 2. Multiply expressions containing surds

Example

$(5+\sqrt{2})(3+\sqrt{2})$

$= 15 + 3\sqrt{2} + 5\sqrt{2} + 2$

$= 17 + 8\sqrt{2}$

2nd example

$(3-2\sqrt{3})(1+4\sqrt{3})$

$= 3 - 2\sqrt{3} + 12\sqrt{3} - 2\times4\times3$

$= 3 + 10\sqrt{3} - 24$

$= -21 + 10\sqrt{3}$

3rd example

$(2+\sqrt{5})(1-\sqrt{3})$

$= 2 + \sqrt{5} - 2\sqrt{3} - \sqrt{15}$

This cannot be simplified further

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TASK 3 Rationalizing Denominators

This involves ensuring that no square root signs are involved in the denominator

Example 1

$$\frac{3}{\sqrt{7}}$$

We can multiply top and bottom by $\sqrt{7}$ without changing its value

$$\frac{3}{\sqrt{7}} = \frac{3}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \frac{3\sqrt{7}}{7}$$

and there is no longer a square root sign in the denominator

In the next two examples we use the factorization of the difference of two squares

$$a^2 - b^2 = (a+b)(a-b)$$

Example 2

$$\frac{3}{5+\sqrt{2}}$$

$$= \frac{3}{(5+\sqrt{2})} \times \frac{(5-\sqrt{2})}{(5-\sqrt{2})}$$

$$= \frac{3(5-\sqrt{2})}{5^2 - \sqrt{2}^2}$$

$$= \frac{15 - 3\sqrt{2}}{25 - 2}$$

$$= \frac{15 - 3\sqrt{2}}{23}$$

Example 3

$$\frac{2+\sqrt{3}}{2\sqrt{3}-1}$$

$$= \frac{(2+\sqrt{3})}{(2\sqrt{3}-1)} \times \frac{(2\sqrt{3}+1)}{(2\sqrt{3}+1)}$$

$$= \frac{(2+\sqrt{3})(2\sqrt{3}+1)}{(2\sqrt{3})^2 - 1^2}$$

$$= \frac{4\sqrt{3} + 2\times 3 + 2 + \sqrt{3}}{4\times 3 - 1}$$

$$= \frac{5\sqrt{3} + 8}{11}$$
