

U61M

Year 13 A-Level Mathematics

Mock Applied Paper 48 Marks

1 Hour

January 2019

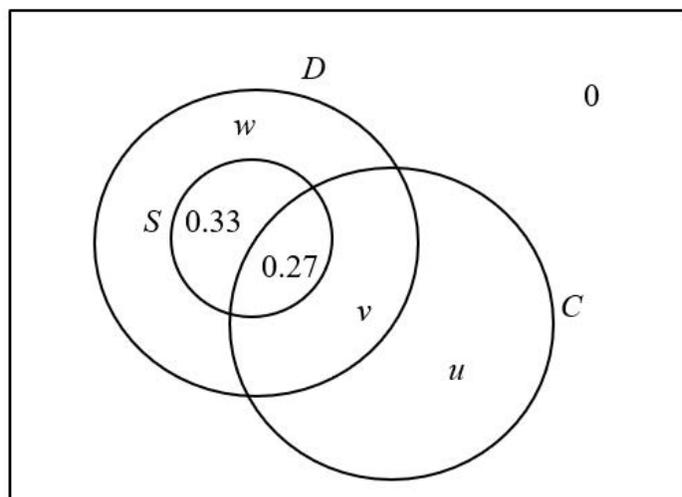
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1. The Venn diagram shows the probabilities of students' lunch boxes containing a drink, sandwiches and a chocolate bar.

D is the event that a lunch box contains a drink,
 S is the event that a lunch box contains sandwiches,
 C is the event that a lunch box contains a chocolate bar,
 u, v and w are probabilities.



- (a) Write down $P(S \cap D')$ (1)

One day, 80 students each bring in a lunch box.
 Given that all 80 lunch boxes contain sandwiches and a drink,

- (b) estimate how many of these 80 lunch boxes will contain a chocolate bar. (3)

Given that the events S and C are independent and that $P(D|C) = \frac{14}{15}$

- (c) calculate the value of u , the value of v and the value of w . (7)

$$a) P(S \cap D') = 0$$

$$b) P(C \setminus (S \cap D)) = \frac{P(C \cap (S \cap D))}{P(S \cap D)}$$

$$= \frac{0.27}{(0.27 + 0.33)} = \frac{0.27}{0.6} = 0.45$$

Estimate for
boxes containing choc

$$= 80 \times 0.45 = \underline{36}$$

$$c) P(D|C) = \frac{14}{15} = \frac{P(D \cap C)}{P(C)}$$

Question 1 continued

$$\frac{14}{15} = \frac{0.27 + V}{0.27 + V + U} \quad (1)$$

S and C independent so

$$P(S \cap C) = P(S) \times P(C)$$

$$0.27 = 0.6 \times (V + U + 0.27)$$

$$\frac{0.27}{0.6} = V + U + 0.27$$

$$0.45 - 0.27 = V + U$$

$$0.18 = V + U \quad (2)$$

Sub for $V + U$ in (1)

$$\frac{14}{15} = \frac{0.27 + V}{0.27 + 0.18}$$

$$\frac{14}{15} = \frac{0.27 + V}{0.45}$$

$$\frac{14}{15} \times 0.45 - 0.27 = V$$

$$\underline{V = 0.15}$$

From (2)

$$U = 0.18 - V$$

$$U = 0.18 - 0.15$$

$$\underline{U = 0.03}$$

$$W = 1 - (0.6 + V + U) = 1 - (0.6 + 0.18) = 0.22$$

$$U = 0.03$$

$$V = 0.15$$

$$W = 0.22$$

(Total for Question 1 is 11 marks)

2. The lifetimes of batteries sold by company X are normally distributed, with mean 150 hours and standard deviation 25 hours.

A box contains 12 batteries from company X .

- (a) Find the expected number of these batteries that have a lifetime of more than 160 hours. (3)

The lifetimes of batteries sold by company Y are normally distributed, with mean 160 hours and 80% of these batteries have a lifetime of less than 180 hours.

- (b) Find the standard deviation of the lifetimes of batteries from company Y . (3)

Both companies sell their batteries for the same price.

- (c) State which company you would recommend. Give reasons for your answer. (2)

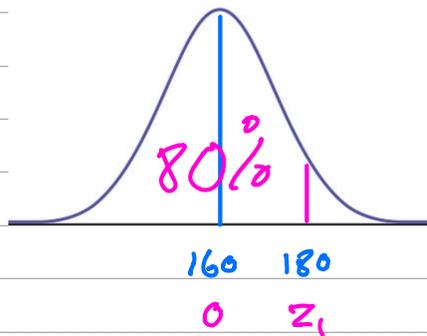
$$a) \quad X \sim N(\overset{\mu}{150}, \overset{\sigma^2}{25^2}) \quad P(X > 160) = 0.34458 \text{ (by calc)}$$

Expected number lasting more than 160 hours

$$= 12 \times 0.34458 = 4.13496$$

$$= 4.13 \quad \text{to 3 s.f.}$$

b)



$$z_1 = \Phi^{-1}(0.8)$$

$$z_1 = 0.841620847$$

$$z = \frac{x - \mu}{\sigma}$$

$$\sigma z = x - \mu$$

$$\sigma = \frac{x - \mu}{z}$$

$$\sigma = \frac{180 - 160}{0.841620847}$$

$$\sigma = 23.76 = 23.8 \text{ to 3 s.f.}$$

Question 2 continued

c) Recommend company Y

Standard deviations are similar but Y

batteries have a greater mean life than

X batteries

(Total for Question 2 is 8 marks)

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

[In this question position vectors are given relative to a fixed origin O .]

3. A particle, P , moves with constant acceleration $(\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2}$.

At time $t = 0$ seconds, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j}) \text{ m}$ and is moving with velocity $\mathbf{u} \text{ m s}^{-1}$.

At time $t = 3$ seconds, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j}) \text{ m}$.

Find \mathbf{u} .

(4)

$$\underline{s} - \underline{s}_0 = \underline{u}t + \frac{1}{2}\underline{a}t^2$$

$$\underline{s} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}t + \frac{1}{2} \begin{pmatrix} 1 \\ -2 \end{pmatrix}t^2$$

$$t = 3 \quad \underline{s} = \begin{pmatrix} -2.5 \\ 8 \end{pmatrix} \quad \begin{pmatrix} -2.5 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}(3) + \frac{1}{2} \begin{pmatrix} 1 \\ -2 \end{pmatrix}(3)^2$$

$$\begin{pmatrix} -4.5 \\ 3 \end{pmatrix} - \begin{pmatrix} 9/2 \\ -9 \end{pmatrix} = 3 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} -9 \\ 12 \end{pmatrix} = 3 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\underline{u} = -3\underline{i} + 4\underline{j} \quad \text{ms}^{-1}$$

Question 3 continued

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(Total for Question 3 is 4 marks)

4.

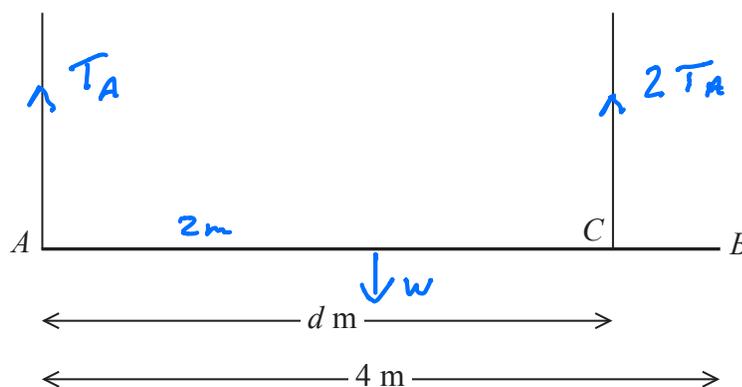


Figure 3

A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A .

- (a) Find the value of d . (6)

A small load of weight kW newtons is attached to the beam at B . The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A .

- (b) Find the value of k . (6)

$$a) \quad \updownarrow \quad 3T_A = W \quad T_C = \frac{W}{3}$$

$$\text{Mom about A} \quad W \times 2 = 2T_A \times d$$

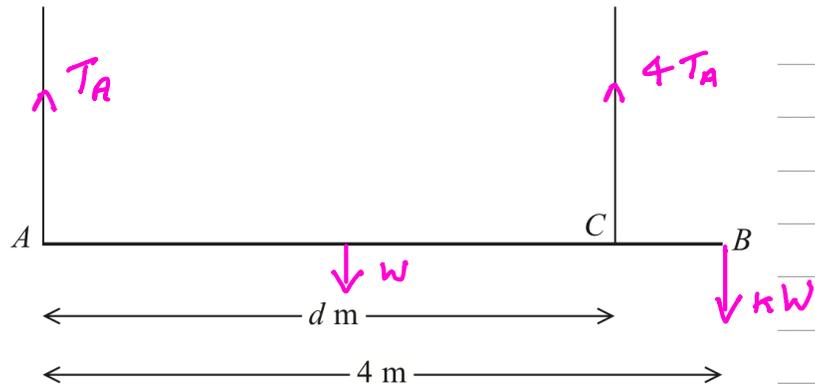
$$2W = \frac{2W}{3} d$$

$$d = 3 \text{ m}$$



Question 4 continued

b)



$$5T_A = W(1+k)$$

$$T_A = \frac{W(1+k)}{5}$$

Moments about A

$$W \times 2 + kW \times 4 = 4T_A \times 3$$

$$2W + 4kW = 4W \frac{(1+k)}{5} \times 3$$

$$W(2 + 4k) = W \times \frac{12(1+k)}{5}$$

$$5(2 + 4k) = 12(1+k)$$

$$10 + 20k = 12 + 12k$$

$$8k = 2$$

$$k = \frac{1}{4}$$

(Total for Question 4 is 12 marks)

5.

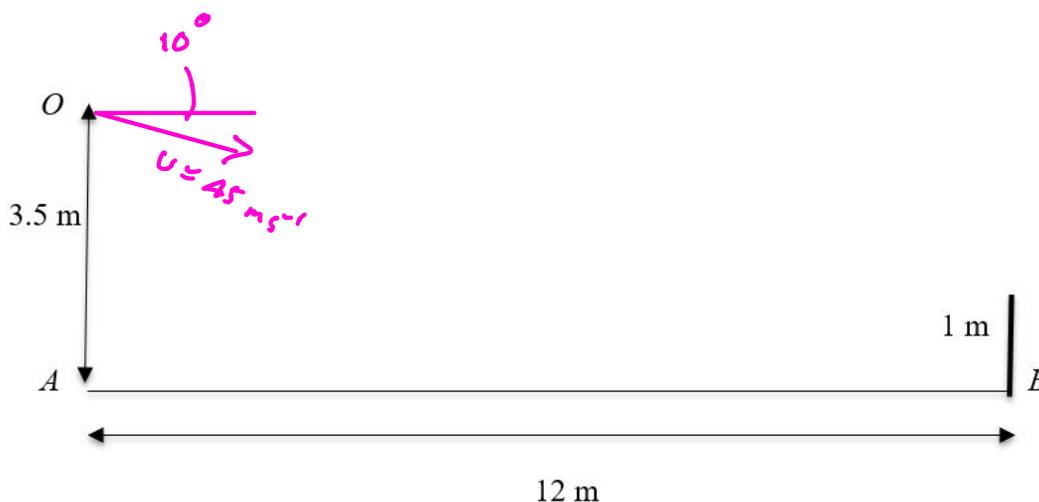


Figure 3

A tennis player serves a ball so as to pass over the net.
The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal.

The ball is struck at a point O which is 3.5 m vertically above the point A which is on horizontal ground.

The bottom of the net is the point B which is on the ground and $AB = 12 \text{ m}$.

The height of the net is 1 m , as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity.

The ball passes over the net at a point which is vertically above B .

Using the model,

(a) find, in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net, (8)

(b) find, to 2 significant figures, the speed of the ball as it passes over the net. (4)

(c) State two limitations of the model that could affect the reliability of your answers. (2)

$$a) \quad u_x = 45 \cos 10 \quad u_y = -45 \sin 10$$

$$\text{Horizontal motion} \quad \text{time to reach net} \quad t = \frac{12}{45 \cos 10}$$

$$t = 0.2707804$$

$$\text{Vertical motion} \quad y - y_0 = ut + \frac{1}{2}at^2$$

$$y - 3.5 = -45 \sin 10 \times 0.2707804 - 4.9 \times 0.2707804^2$$

Question 5 continued

$$y = 1.0248 \text{ m}$$

so 2.5 cm above net to 2 s.f.

b) $v_x = u_x = 45 \cos 10 = 44.3163 \text{ ms}^{-1}$

$$v_y = u_y + at = -45 \sin 10 - 9.8 \times 0.2707804$$

$$= -10.4678 \text{ ms}^{-1}$$

$$\text{Speed} = \sqrt{44.3163^2 + (-10.4678)^2}$$

$$= 45.5358$$

$$= 46 \text{ ms}^{-1} \quad \text{to 2 s.f.}$$

c) air resistance, size of ball, spin on ball
