Intro to Probability

Scale

unlikely 50:50 likely

0 (impossible) — $\frac{1}{2}$ (0·5, 50%) — 1 (certain)

Record probabilities as fractions, decimals or percentages

Dice, Coins, Playing Cards

Dice	Coins	Playing Cards
1	H	52 cards: 13 Red Diamonds
2	T	13 Red Hearts
3		13 Black Spades
4		13 Black Clubs
5		A 2 3 4 5 6 7 8 9 10 J Q K
6		

Roll a die

$P(1) = \frac{1}{6}$

$P(2) = \frac{1}{6}$

$P(3) = \frac{1}{6}$

$P(4) = \frac{1}{6}$

$P(5) = \frac{1}{6}$

$P(6) = \frac{1}{6}$

Coin

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

Sample Space

1	2	3
6	5	4

possible outcomes from rolling a die

Event A roll a 2

Event B roll a 3

are said to be <u>mutually exclusive events</u>

If mutually exclusive then the probability of A or B happening written as $P(A \cup B)$ is given by $P(A \cup B) = P(A) + P(B)$

In this case $P(A \cup B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

Two events C and D are said to be independent if the probability they both happen written as $P(C \cap D)$ is given by

$$P(C \cap D) = P(C) \times P(D)$$

This is the test for independence

Example Roll a die and spin a coin

Let A be event roll a 4

Let B be event spin a head

$P(A) = \frac{1}{6}$ $\qquad P(B) = \frac{1}{2}$

$P(A \cap B)$ possible equally likely outcomes

H1	T1
H2	T2
H3	T3
H4	T4
H5	T5
H6	T6

$= \frac{1}{12}$

$$P(A) \times P(B) = P(A \cap B)$$

$$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

$\therefore$ independent

If mutually exclusive events cover the whole sample space they are said to be <u>exhaustive</u>.

Ex 2 Rolling a die.

Let A be event an even number

$$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$$

Let B be event number greater than 3

$$\therefore P(B) = \frac{3}{6} = \frac{1}{2}$$

Are A and B independent?

1, 2, 3, 4, 5, 6

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$ $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{3}$$

$\therefore$ not independent (or can say dependent)

Conditional Probability (A2 syllabus)

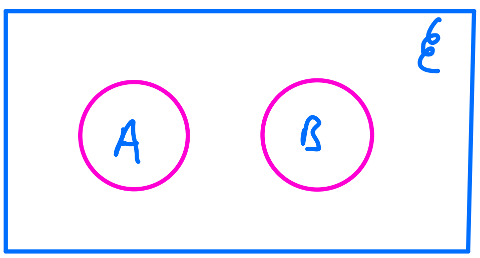
$P(A|B)$ means the probability of A given that B has happened

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

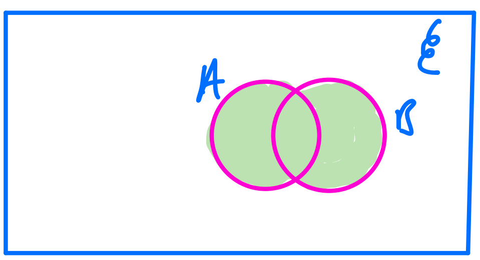
Notice that if A and B are independent

$$P(A|B) = \frac{P(A) \times \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

Venn Diagrams

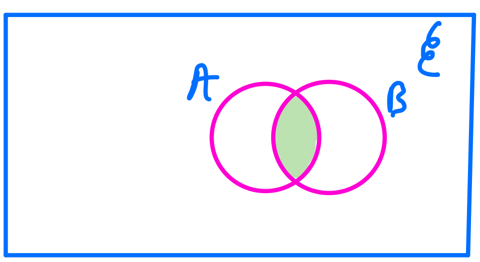


A and B are mutually exclusive



$A \cup B$ union

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$A \cap B$ intersection

Ex 3

Cards Let A be event pick a club

Let B be event pick a 5

Are they independent when picking one card

$P(A) = \frac{13}{52} = \frac{1}{4}$

$P(A) \times P(B) = P(A \cap B)$

$P(B) = \frac{4}{52} = \frac{1}{13}$

$\frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$ ✓

$P(A \cap B) = \frac{1}{52}$

These events are independent

Let C be event a black card

Are A and C independent?

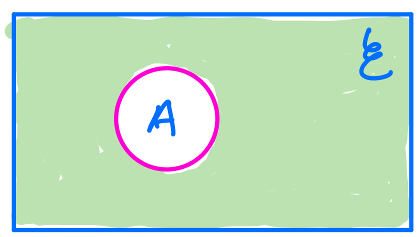
$P(A) = \frac{1}{4}$

$P(A \cap C) = \frac{13}{52} = \frac{1}{4}$

$P(C) = \frac{26}{52} = \frac{1}{2}$

$\frac{1}{4} \times \frac{1}{2} \neq \frac{1}{4}$

A and C are not independent



not A or A complement

written as A' or A^c