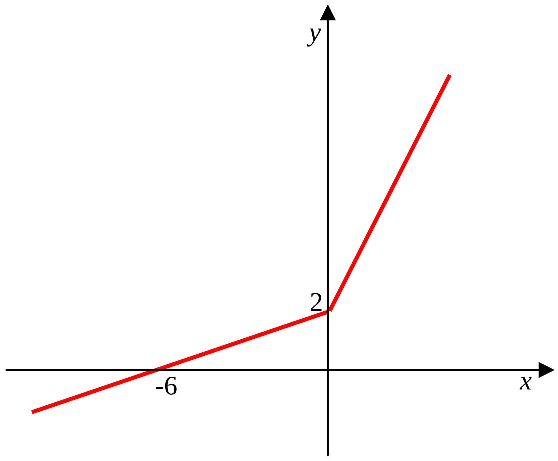
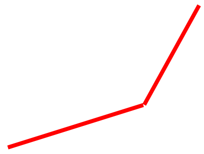
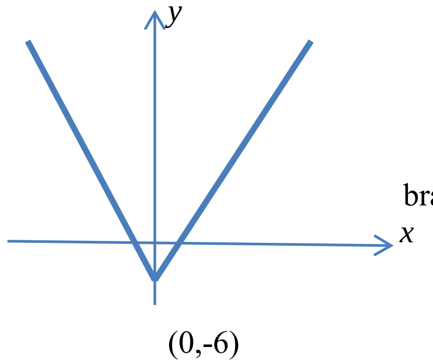
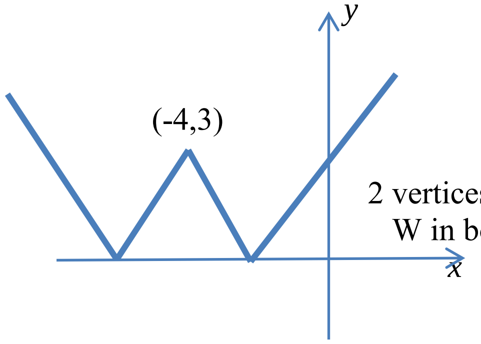
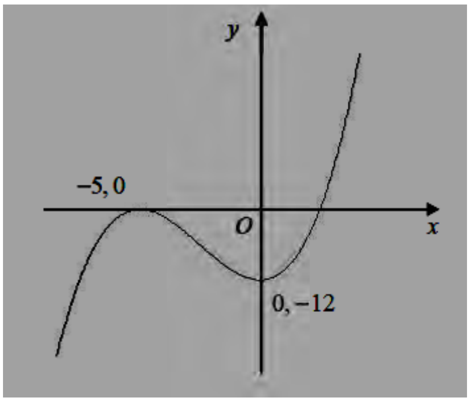
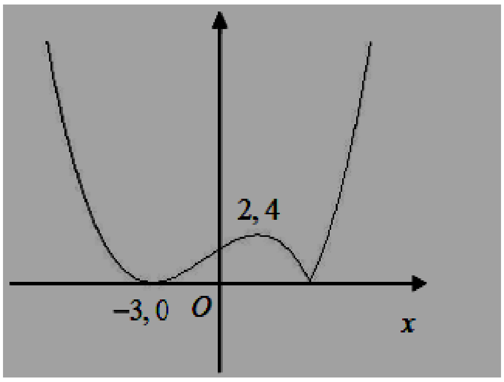
Question Number	Scheme		Marks
6.			
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore \mathrm{f}^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\underline{\dfrac{3+5x}{x+2}}$	A1 oe
			(3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$	<u>Correct Range</u>	B1 (1)
(c)		Deduces that $\mathrm{g}(2)$ is 0. Seen or implied.	M1
	g g(2)= g (0) $= -6$, from sketch.	-6	A1 (2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1
			(2)

Question Number	Scheme		Marks
(e)(ii)		Correct shape 	B1
		Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1
			(4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$
			(1) **[13]**

Question Number	Scheme	Marks
3 (a)		
	V shape	B1
	vertex on y axis &both branches of graph cross x axis	B1
	„y" co-ordinate of R is -6	B1
		(3)
(b)		
	W shape	B1
	2 vertices on the negative x axis. W in both quad 1 & quad 2.	B1dep
	R"=(-4,3)	B1
		(3)
		6 Marks
4 (a)	$y = 4 - \ln(x+2)$ $\ln(x+2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$	M1
	$f^{-1}(x) = e^{4-x} - 2$ oe	M1A1
		(3)
(b)	$x \leq 4$	B1
		(1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$	M1
	$fg(x) = 4 - x^2$	dM1A1
		(3)
(d)	$fg(x) \leq 4$	B1ft
		(1)
		8 Marks

Question No	Scheme	Marks
2	(a)  Shape *x* coordinates correct y coordinates correct	B1 B1 B1 (3)
	(b)  Shape Max at (2,4) Min at (-3,0)	B1 B1 B1 (3)
		6 marks

(a)

B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross *x* axis.

B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the *y* axis (marked -12).

B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch ×3 parallel to the *y* axis. The maximum P' must be on the *x* axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

(b)

B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.

B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum.
Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.

B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

Question No	Scheme	Marks
7	(a) $2x^2+7x-4=(2x-1)(x+4)$	B1
	$\dfrac{3(x+1)}{(2x-1)(x+4)}-\dfrac{1}{(x+4)}=\dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
	$=\frac{x+4}{(2x-1)(x+4)}$	M1
	$=\frac{1}{2x-1}$	A1*
		(4)
	(b) $y=\frac{1}{2x-1}\Rightarrow y(2x-1)=1\Rightarrow 2xy-y=1$	
	$2xy=1+y\Rightarrow x=\dfrac{1+y}{2y}$	M1M1
	$y\ OR\ f^{-1}(x)=\dfrac{1+x}{2x}$	A1
		(3)
	(c) x>0	B1
		(1)
	(d) $\frac{1}{2\ln(x+1)-1}=\frac{1}{7}$	M1
	$\ln(x+1)=4$	A1
	$x=e^4-1$	M1A1
		(4)
		12 Marks

(a)

B1 Factorises the expression $2x^2+7x-4=(2x-1)(x+4)$. This may not be on line 1

M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$,Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct.
You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

For example $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

Whereas $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

(b)

M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2x-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2x-1 = \frac{1}{y} \to x = \frac{\frac{1}{y} \pm 1}{2}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2xy - y = 1 \to 2xy = 1 \pm y \to x = \frac{1 \pm y}{2y}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to 2x-1 = \frac{1}{y} \to 2x = \frac{1}{y} + 1 \to x = \frac{1}{2y} + 1$ (allow slip on ÷2)

A1 Must be written in terms of x but can be $y = \frac{1+x}{2x}$ or equivalent inc $y = \frac{\frac{1}{x}+1}{2}$, $y = \frac{x^{-1}+1}{2}$, $y = \frac{1}{2x} + \frac{1}{2}$

(c)

B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x≥ 0 , y>0, [0, ∞], $f^{-1}(x) > 0$

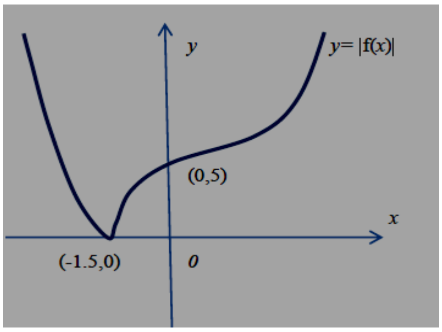
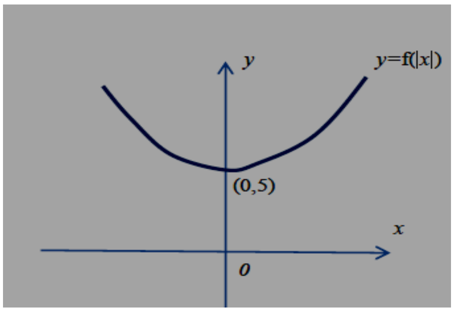
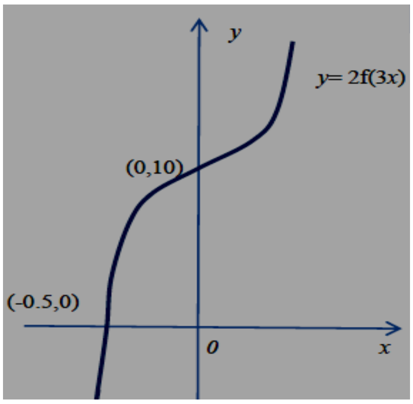
(d)

M1 Attempt to write down fg(x) and set it equal to 1/7.
The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2lnx+1-1} = \frac{1}{7}$

A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $ln(x+1)^2 = 8$

M1 Moving from $\ln(x \pm A) = c \quad A \neq 0 \ to \ x =$ The ln work must be correct
Alternatively moving from $ln(x+1)^2 = c$ to $x = \cdots$
Full solutions to calculate x leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

A1 Correct answer only $= e^4 - 1$. Accept $e^4 - e^0$

Question Number	Scheme		Marks
4.(a)		Shape including cusp	B1
		(-1.5, 0) **and** (0, 5)	B1
			(2)
(b)		Shape	B1
		(0,5)	B1
			(2)
(c)		Shape	B1
		(0,10)	B1
		(-0.5, 0)	B1
			(3)
			(7 marks)

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the *x*-axis at (-1.5, 0) **and** crossing the *y*-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the *y*- axis

B1 (0,5) lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

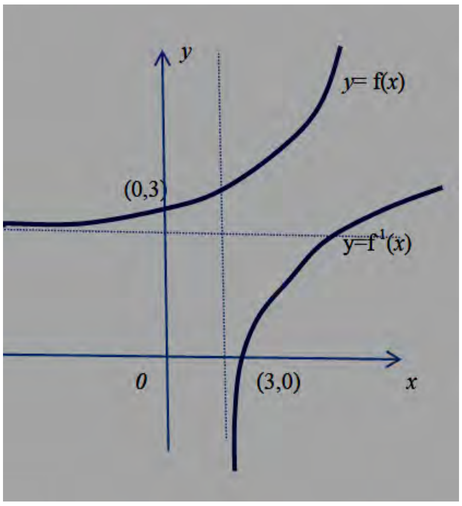
B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to f(*x*)

B1 Curve **crosses** the *y* axis at (0, 10). The curve must appear in both quadrants 1 and 2

B1 Curve **crosses** the *x* axis at (-0.5, 0). The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using (0,3) as an example, accept both (3,0) or 3 written on the *y* axis (as long as the curve passes through the point)

Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1 Otherwise follow scheme

Question Number	Scheme	Marks
6.	(a) f(x)>2	B1 (1)
	(b) $\text{fg}(x) = e^{\ln x} + 2, = x + 2$	M1,A1 (2)
	(c) $e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$	M1A1
	$\Rightarrow 2x + 3 = \ln 4$	
	$\Rightarrow x = \dfrac{\ln 4 - 3}{2}$ or $\ln 2 - \dfrac{3}{2}$	M1A1 (4)
	(d) Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y-2) = x$	M1
	$\text{f}^{-1}(x) = \ln(x-2), \quad x > 2.$	A1 , **B1ft** (3)
	(e) 	
	Shape for f(x)	B1
	(0, 3)	B1
	Shape for $\text{f}^{-1}(x)$	B1
	(3, 0)	B1 (4)
		(14 marks)

(a) B1 Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x>2

(b) M1 For applying the correct order of operations. Look for $e^{\ln x} + 2$. Note that $\ln e^x + 2$ is M0
A1 Simplifies $e^{\ln x} + 2$ to $x + 2$. Just the answer is acceptable for both marks

(c) M1 Starts with $e^{2x+3} + 2 = 6$ and proceeds to $e^{2x+3} = ...$
A1 $e^{2x+3} = 4$
M1 Takes ln's both sides, $2x + 3 = \ln ..$ and proceeds to x=….
A1 $x = \dfrac{\ln 4 - 3}{2}$ oe. eg $\ln 2 - \dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) **Note that this is marked M1A1A1 on EPEN**

M1 Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject.
All ln work must be correct. The 2 must be dealt with first.
Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0

A1 $f^{-1}(x) = \ln(x-2)$ or $y = \ln(x-2)$ or $y = \ln|x-2|$ There must be some form of bracket

B1ft Either $x>2$, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$
Do not accept y>2 or $f^{-1}(x)>2$.

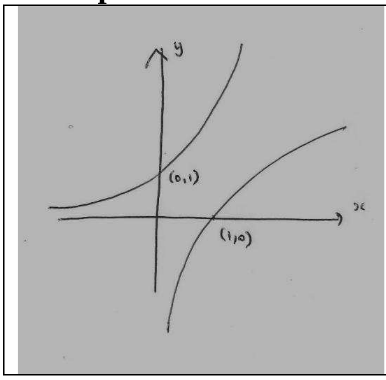
(e) B1 Shape for $y=e^x$. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the *x* axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.

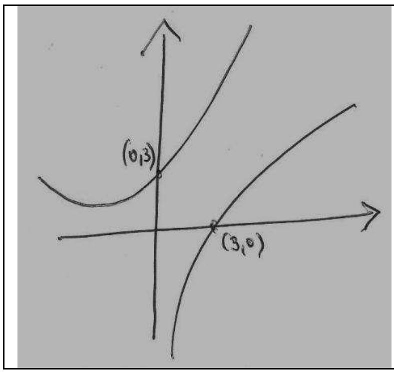
B1 (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

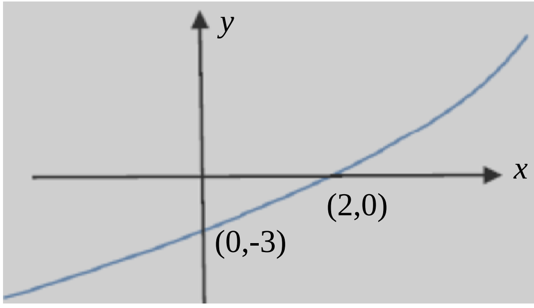
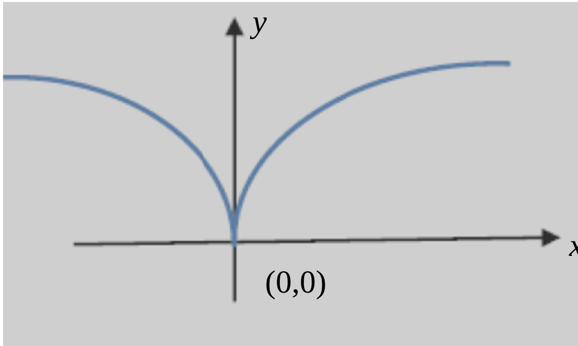
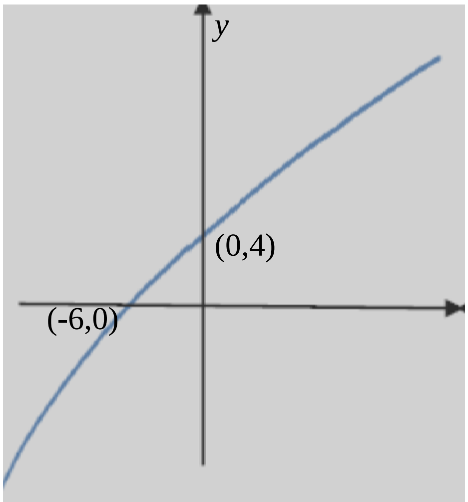
B1 Shape for $y=\ln x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the *y* axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects $y=e^x$

B1 (3, 0) lies on the curve. Accept 3 written on the *x* axis as long as the point lies on the curve

Condone lack of labels in this part
Examples

	Scores **1,0,1,0.** Both shapes are fine, do not be concerned about asymptotes appearing at x=2, y=2. (See notes) Both co-ordinates are incorrect

	Scores **0,1,1,1** Shape for $y = e^x$ is incorrect, there is a minimum point on the graph. All other marks an be awarded

Question Number	Scheme		Marks
3.	(a) ff(-3)= f(0),=2		M1,A1
			(2)
	(b)  $y = \mathrm{f}^{-1}(x)$	Shape	B1
		(0,-3) and (2,0)	B1
			(2)
	(c)  y=f(|x|)-2	Shape	B1
		(0,0)	B1
			(2)
	(d)  x	Shape	B1
		(-6,0) or (0,4)	B1
		(-6,0) and (0,4)	B1
			(3)
			(9 marks)

(a) M1 A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not.

Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points.

A1 Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b)

B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum

B1 This is independent to the first mark and for the graph passing through (0,-3) and (2, 0)

Accept -3 and 2 marked on the correct axes.
Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes
Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P'and Q' are marked in the correct place on the graph
There must be a graph for this to be awarded

(c)

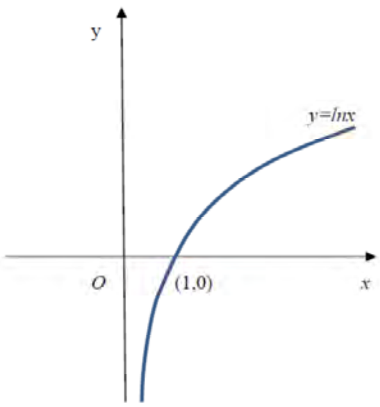
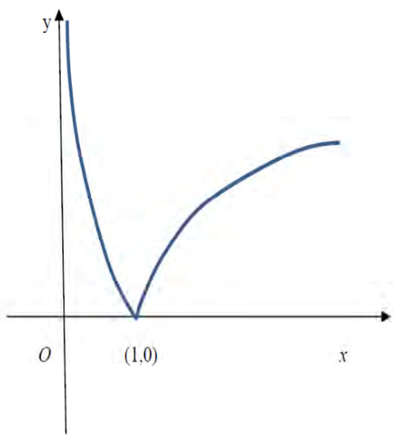
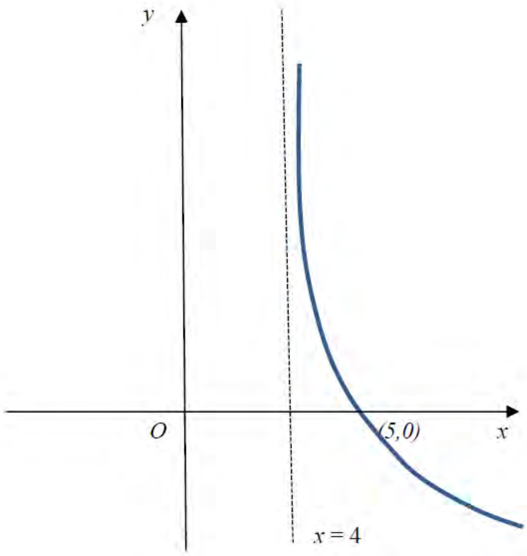
B1 Award for a correct shape 'roughly' symmetrical about the *y*- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum

B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked

(d) B1 Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.

B1 The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations

B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

Question Number	Scheme		Marks
2(i)		*ln* graph crossing *x* axis at (1,0) and asymptote at *x*=0	B1
2(ii)		Shape including cusp	B1ft
		Touches or crosses the *x* axis at (1,0)	B1ft
		Asymptote given as *x*=0	B1
2(iii)		Shape	B1
		Crosses at (5, 0)	B1ft
		Asymptote given as *x*=4	B1
			(7 marks)

Notes for Question 2

(i) B1 Correct shape, correct position and passing through (1, 0).
Graph must 'start' to the rhs of the y - axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone 'slips'. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. **Do not with hold this mark if (x=0) the asymptote is incorrect or not given**.

(ii) B1ft Correct shape **including the cusp** wholly contained in quadrant 1.
The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum.. The shape to the lhs of the cusp should not bend backwards past (1,0)
Tolerate a 'linear' looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items.
Follow through on an incorrect sketch in part (i) as long as it was above and below the x axis.

B1ft The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1' on the x axis. Condone the point marked on the correct axis as (0, 1)
Follow through on an incorrect intersection in part (i).

B1 Award for the asymptote to the curve given/ marked as x = 0. Do not allow for it given/ marked as 'the y axis'. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at x = 0. Accept if x=0 is drawn separately to the y axis.

(iii)

B1 Correct shape.
The gradient should always be negative and becoming less steep. It must be approximately infinite at the *lh* end and not have an obvious minimum. The lh end must not bend 'forwards' to make a C shape. The position is not important for this mark. See practice and qualification for clarification.

B1ft The graph crosses (or touches) the x axis at (5, 0). Allow for the curve passing through a point marked '5' on the x axis. Condone the point marked on the correct axis as (0, 5)
Follow through on an incorrect intersection in part (i). Allow for $\left((i)+4,0\right)$

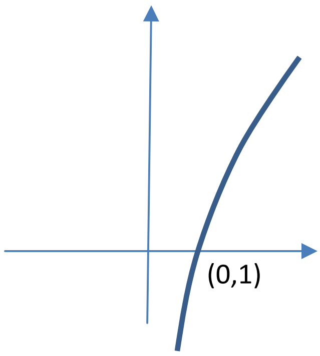
B1 The asymptote is given/ marked as x = 4. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the y axis).

If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.

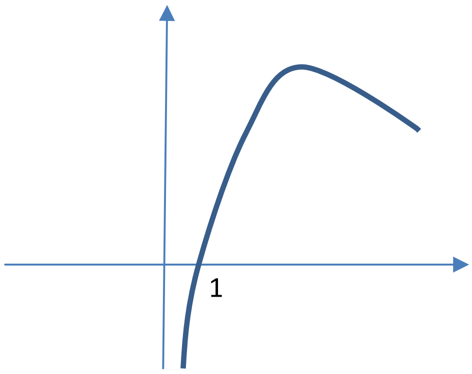
<u>Examples of graphs in number 2</u>

Part (i)

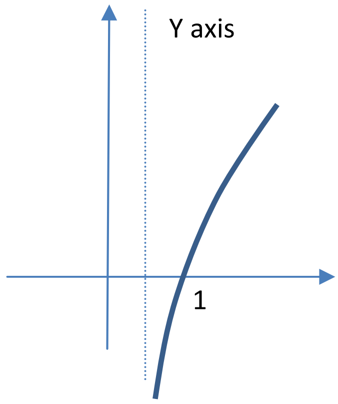
Condoned



Not condoned

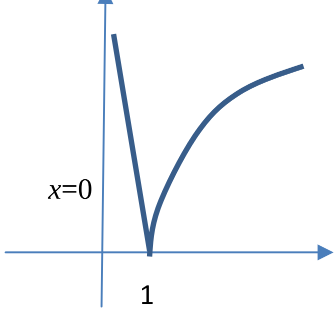


Condone

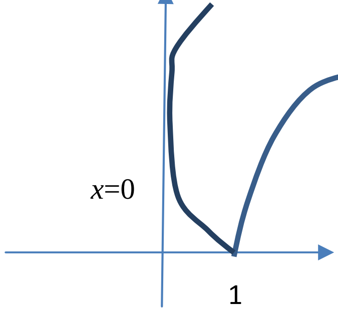


Part (ii)

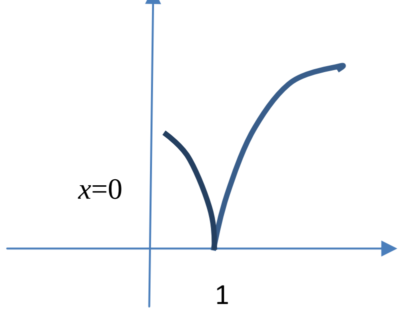
1,1,1



0,1,1,

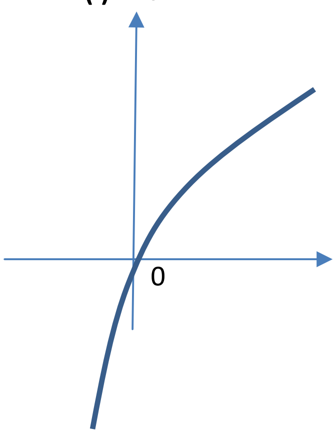


0,1,0,

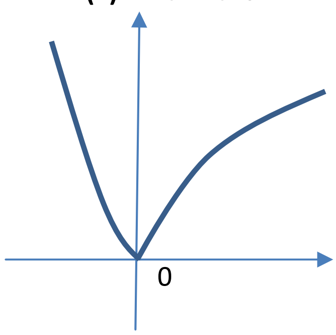


Example of follow through in part (ii) and (iii)

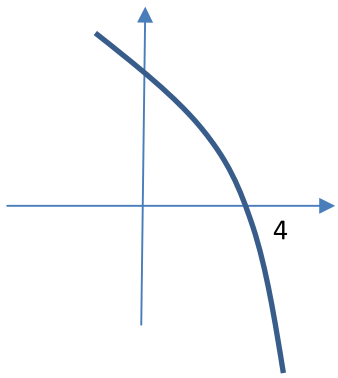
(i) B0



(ii) B1ftB1ftB0



(iii) B0B1ftB0



Question Number	Scheme	Marks
7(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	ff(0) = f(5), = 3	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y-4 = xy+3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5-2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to 7(d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2$ *or* $5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = ...$	M1
	$\Rightarrow x = 0.4$ oe	A1 **(5)**

Notes for Question 7

(a)

B1 Correct range. Allow $0 \leqslant f(x) \leqslant 10$, $0 \leqslant f \leqslant 10$, $0 \leqslant y \leqslant 10$, $0 \leqslant \text{range} \leqslant 10$, $[0,10]$

Allow $f(x) \geqslant 0$ and $f(x) \leqslant 10$ but not $f(x) \geqslant 0$ or $f(x) \leqslant 10$

Do Not Allow $0 \leqslant x \leqslant 10$. The inequality must include BOTH ends

(b)

B1 For correct one application of the function at x=0

Possible ways to score this mark are f(0)=5, f(5) $0 \to 5 \to \ldots$

B1: 3 ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1 For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting y = g(x), multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs

dM1 Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this mark

A1 Correct answer. No need to state the domain. Allow $g^{-1}(x) = \dfrac{5x-4}{3+x}$ $y = \dfrac{5x-4}{3+x}$

Accept alternatives such as $y = \dfrac{4-5x}{-3-x}$ and $y = \dfrac{5-\frac{4}{x}}{1+\frac{3}{x}}$

(d)

M1 Stating or implying that $f(x) = g^{-1}(16)$. For example accept $\dfrac{4+3f(x)}{5-f(x)} = 16 \Rightarrow f(x) = ..$

A1 Stating $f(x) = 4$ or implying that solutions are where $f(x) = 4$

B1 x = 6 and may be given if there is no working

M1 Full method to obtain other value from line $y = 5-2.5x$

$5-2.5x = 4 \Rightarrow x = \ldots$.

Alternatively this could be done by similar triangles. Look for $\dfrac{2}{5} = \dfrac{2-x}{4}\,(oe) \Rightarrow x = ..$

A1 0.4 or 2/5

Alt 1 to (d)

M1 Writes $gf(x) = 16$ with a linear $f(x)$. The order of gf(x) must be correct

Condone invisible brackets. Even accept if there is a modulus sign.

A1 Uses $f(x) = x-2$ or $f(x) = 5-2.5x$ in the equation $gf(x) = 16$

B1 x = 6 and may be given if there is no working

M1 Attempt at solving $\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$. The bracketing must be correct and there must be no more than one error in their calculation

A1 $x = 0.4, \dfrac{2}{5}$ or equivalent