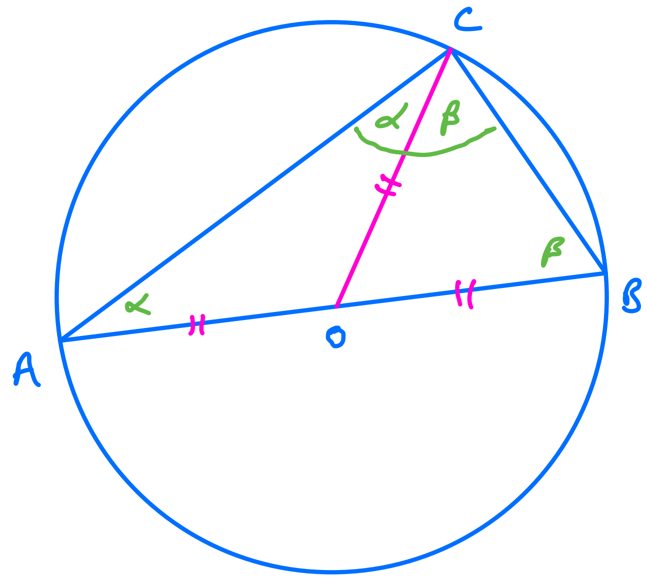
Circle Theorem Proofs

1) The angle in a semi-circle is 90°

<u>Proof</u>



Join OC to form two isosceles Δs

Let $\angle OCA = \alpha$ Let $\angle OCB = \beta$

$\Rightarrow \angle OAC = \alpha$ $\Rightarrow \angle OBC = \beta$

(base $\angle$s of isos Δs)

$\angle AOC = 180 - 2\alpha$ ($\angle$ sum of Δ)

$\angle BOC = 180 - 2\beta$ ($\angle$ sum of Δ)

But $\angle AOC + \angle BOC$ is a straight line $= 180^\circ$

$\therefore \quad 180 - 2\alpha + 180 - 2\beta = 180$

$180 + 180 - 180 = 2\alpha + 2\beta$

$180 = 2\alpha + 2\beta$

$\div \quad 90 = \alpha + \beta$

$\therefore \angle ACB = 90^\circ$

$\therefore$ an angle in a semi-circle $= 90^\circ$

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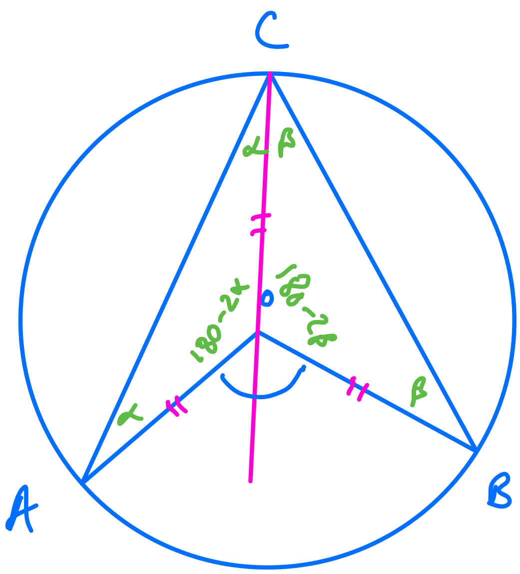
2) The angle subtended at the centre by an arc or chord is twice the angle subtended at the circumference

<u>Proof</u> (limited to simplest case)



Join and extend OC to form two isosceles Δs

Let $\angle ACO = \alpha$ and $\angle BCO = \beta$

$\Rightarrow \angle OAC = \alpha$ and $\angle OBC = \beta$

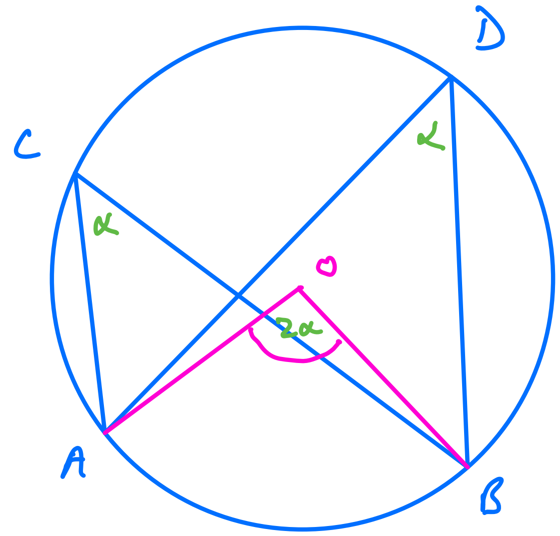
$\angle AOC = 180 - 2\alpha$ ($\angle$ sum of Δ)
$\angle BOC = 180 - 2\beta$ ($\angle$ sum of Δ)

$$\begin{aligned}\angle AOB &= 360^\circ - (180 - 2\alpha) - (180 - 2\beta) \\ &= 360 - 180 + 2\alpha - 180 + 2\beta \\ &= 2\alpha + 2\beta \\ &= 2(\alpha + \beta) \\ &= 2 \times \angle ACB\end{aligned}$$

Angle at centre twice angle at circumference

3) Angles in the same segment are equal

Proof



Let $ACB = \alpha$

Join OA and OB

then $\angle AOB = 2\alpha$
($\angle$ at centre twice $\angle$ at circumference)

$\Rightarrow \angle ADB = \alpha$
($\angle$ at centre twice $\angle$ at circumference)

$\therefore \angle ACB = \angle ADB$

Angles in the segment are equal

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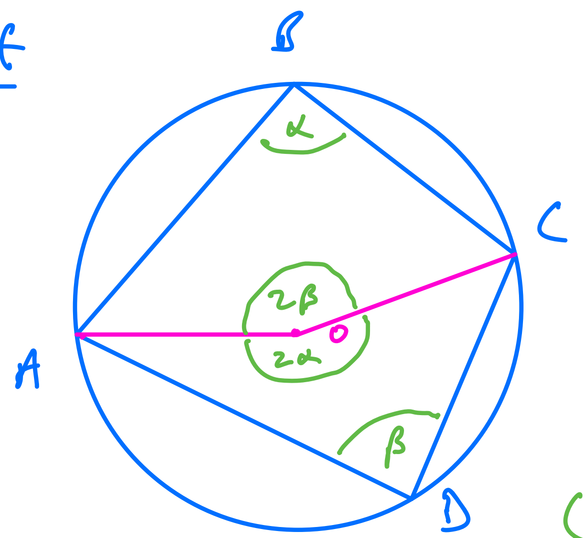
4) Opposite angles of a cyclic quadrilateral add up to 180° (are supplementary)

Proof



Let $\angle ABC = \alpha$

Let $\angle ADC = \beta$

Label centre O

Join OA and OC

$\angle AOC$ (major) $= 2\alpha$
$\angle AOC$ (minor) $= 2\beta$
($\angle$ at centre twice $\angle$ at circ)

$$2\alpha + 2\beta = \text{full circle} = 360^\circ$$

$$\Rightarrow \alpha + \beta = 180^\circ$$

Opposite angles of a cyclic quad add up 180°

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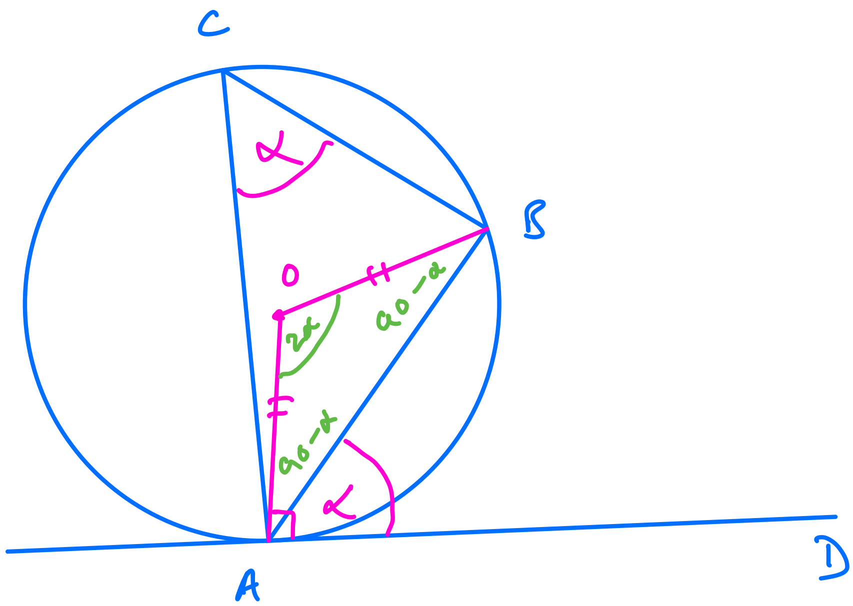
5) Alternate Segment Theorem

The angle between a tangent and a chord is equal to an angle made by the chord in the alternate segment



Proof Join OA and OB to form radii

Then $\angle DAO = 90^\circ$ (tangent-radius)

$\Rightarrow BAO = 90 - \alpha$

$\Rightarrow ABO = 90 - \alpha$ (isos △)

$\therefore \angle AOB = 180 - (90 - \alpha) - (90 - \alpha)$

$\angle AOB = 180 - 90 + \alpha - 90 + \alpha$

$\angle AOB = 2\alpha$

$\therefore \angle ACB = \alpha$ ($\angle$ at centre is twice $\angle$ at the circ)