NON-LINEAR SIMULTANEOUS EQUATIONS — EXERCISE

Solve:

1) $\begin{cases} y = x^2 - 3x + 9 \\ y = x^2 + 5x - 7 \end{cases}$

2) $\begin{cases} y = x^2 + 4x + 14 \\ y = 2x^2 + 3x + 2 \end{cases}$

3) $\begin{cases} y = x^2 - 3x + 5 \\ y = 3x + 12 \end{cases}$

4) $\begin{cases} y = x^2 + 3x - 3 \\ y = x + 1 \end{cases}$ Answers in surd form

5) $\begin{cases} y = x^2 + 5x + 5 \\ y = 3x + 1 \end{cases}$ Prove graphs do not intersect

6) $\begin{cases} x^2 + y^2 = 125 \\ y = 2x \end{cases}$

7) $\begin{cases} y = \dfrac{8}{x+1} \\ y = x + 3 \end{cases}$

NON-LINEAR SIMULTANEOUS EQUATIONS — EXERCISE

1) $y = x^2 - 3x + 9$ ①

$y = x^2 + 5x - 7$ ②

Subst for y in ①

$x^2 + 5x - 7 = x^2 - 3x + 9$

$x^2 + 5x - 7 - x^2 + 3x - 9 = 0$

$8x - 16 = 0$

$8x = 16$

$x = 2$

Subst for x in ①

$y = 2^2 - 3(2) + 9$

$y = 4 - 6 + 9$

$y = 7$

Solution:

$x = 2, \; y = 7$

2) $y = x^2 + 4x + 14$ ①

$y = 2x^2 + 3x + 2$ ②

Subst for y in ①

$2x^2 + 3x + 2 = x^2 + 4x + 14$

$2x^2 + 3x + 2 - x^2 - 4x - 14 = 0$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$\Rightarrow \quad x = 4 \quad \text{or} \quad x = -3$

when $x = 4$, $y = 4^2 + 4(4) + 14$

$y = 16 + 16 + 14$

$y = 46$

$\therefore \quad \underline{x = 4, \; y = 46}$

when $x = -3$, $y = (-3)^2 + 4(-3) + 14$

$y = 9 - 12 + 14$

$y = 11$

$\therefore \quad \underline{x = -3, \; y = 11}$

Solution: $x = 4, \; y = 46$

$x = -3, \; y = 11$

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3) $y = x^2 - 3x + 5$ ①

$y = 3x + 12$ ②

Subst for y in ①

$3x + 12 = x^2 - 3x + 5$

$0 = x^2 - 3x + 5 - 3x - 12$

$0 = x^2 - 6x - 7$

$0 = (x - 7)(x + 1)$

$\Rightarrow \quad x = 7$ or $x = -1$

When $x = 7$, $y = 3(7) + 12$

$y = 33$

$\therefore \underline{x = 7, \; y = 33}$

When $x = -1$, $y = 3(-1) + 12$

$y = 9$

$\therefore \underline{x = -1, \; y = 9}$

Solution:

$x = 7, \quad y = 33$

$x = -1, \quad y = 9$

4) $y = x^2 + 3x - 3$ ①

$y = x + 1$ ②

Subst for y in ②

$x^2 + 3x - 3 = x + 1$

$x^2 + 3x - 3 - x - 1 = 0$

$x^2 + 2x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 + 16}}{2}$$

$$x = \frac{-2 \pm \sqrt{20}}{2}$$

$$x = \frac{-2 \pm \sqrt{4 \times 5}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{5}}{2}$$

$$x = -1 \pm \sqrt{5}$$

When $x = -1 + \sqrt{5}$ $\quad y = -1 + \sqrt{5} + 1$

$y = \sqrt{5}$

When $x = -1 - \sqrt{5}$ $\quad y = -1 - \sqrt{5} + 1$

$y = -\sqrt{5}$

Solution:

$x = -1 + \sqrt{5}, \; y = \sqrt{5}$

$x = -1 - \sqrt{5}, \; y = -\sqrt{5}$

NON-LINEAR SIMULTANEOUS EQUATIONS — EXERCISE

5) $y = x^2 + 5x + 5$ ①

$y = 3x + 1$ ②

Subst for y in ②

$x^2 + 5x + 5 = 3x + 1$

$x^2 + 5x + 5 - 3x - 1 = 0$

$x^2 + 2x + 4 = 0$

Consider discriminant $b^2 - 4ac$

$b^2 - 4ac$

$= 2^2 - 4 \times 1 \times 4$

$= 4 - 16$

$= -12$

Since discriminant < 0

there are no real roots

and so no points of

intersection

6) $x^2 + y^2 = 125$ ①

$y = 2x$ ②

Subst for y in ①

$x^2 + (2x)^2 = 125$

$x^2 + 4x^2 = 125$

$5x^2 = 125$

$x^2 = \frac{125}{5}$

$x^2 = 25$

$\Rightarrow x = \pm 5$

when $x = 5$, $y = 2(5) = 10$

when $x = -5$, $y = 2(-5) = -10$

Solution:

$x = 5, \quad y = 10$

$x = -5, \quad y = -10$

7) $y = \frac{8}{x+1}$ ①

$y = x+3$ ②

Subst for y in ①

$$x+3 = \frac{8}{x+1}$$

$$(x+3)(x+1) = 8$$

$$x^2+3x+x+3 = 8$$

$$x^2+4x+3-8 = 0$$

$$x^2+4x-5 = 0$$

$$(x+5)(x-1) = 0$$

$\Rightarrow x = -5$ or $x = 1$

When $x = -5$, $y = -5+3$

$y = -2$

$\therefore$ $x = -5$, $y = -2$

When $x = 1$, $y = 1+3$

$y = 4$

$\therefore$ $x = 1$, $y = 4$

Solution:

$x = -5$, $y = -2$

$x = 1$, $y = 4$