

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$


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### Exercise 13A

If  $\frac{dy}{dx} = 4x^{\frac{1}{2}} \Rightarrow y = \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + C$

$$y = \frac{8x^{\frac{3}{2}}}{3} + C$$


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| L  $\frac{dy}{dx} = -14x^{-8} \Rightarrow y = \frac{-14x^{-7}}{-7} + C$

$$y = 2x^{-7} + C$$


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2c)  $\frac{dy}{dx} = 4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

$$\Rightarrow y = 4x - \frac{12x^{-3}}{-3} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + C$$


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2f)  $\frac{dy}{dx} = 5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

$$\Rightarrow y = \frac{5x^5}{5} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - \frac{12x^{-4}}{-4} + C$$

$$y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + C$$

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$$3c) f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$

$$\Rightarrow f(x) = \frac{\frac{1}{2}x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{\frac{1}{2}x^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + C$$

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$$3f) f'(x) = 9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$$

$$\Rightarrow f(x) = \frac{9x^3}{3} + \frac{4x^{-2}}{-2} + \frac{\frac{1}{4}x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$f(x) = 3x^3 - 2x^{-1} + \frac{1}{8}x^{\frac{1}{2}} + C$$

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### Exercise 13B

$$6a) \int \left( \frac{2x^3+3}{x^2} \right) dx = \int (2x + 3x^{-2}) dx$$
$$= \frac{2x^2}{2} + \frac{3x^{-1}}{-1} + C$$
$$= x^2 - 3x^{-1} + C$$

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$$7a) f(x) = \left( x + \frac{1}{x} \right)^2 = \left( x^2 + 2 + \frac{1}{x^2} \right)$$

$$\int f(x) dx = \int \left( x^2 + 2 + \frac{1}{x^2} \right) dx = \int (x^2 + 2 + x^{-2}) dx$$
$$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + C$$

$$= \frac{1}{3}x^3 + 2x - x^{-1} + C$$

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$$\begin{aligned} 8d) \quad \int \frac{(2x+1)^2}{\sqrt{x}} dx &= \int \left( \frac{4x^2 + 4x + 1}{\sqrt{x}} \right) dx \\ &= \int \left( 4x^{3/2} + 4x^{1/2} + x^{-1/2} \right) dx \\ &= \frac{4x^{5/2}}{5/2} + \frac{4x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C \\ &= \frac{8}{5}x^{5/2} + \frac{8}{3}x^{3/2} + 2x^{1/2} + C \end{aligned}$$

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