

Vol of Revolution

Exercise 5B

Q3

a)
$$R = \int_4^9 x \, dy = \int_4^9 \left(\sqrt{y} + \frac{1}{y^2} \right) dy$$

$$= \left[\frac{y^{3/2}}{3/2} + \frac{y^{-1}}{-1} \right]_4^9$$

$$= \left[\frac{2}{3} y^{3/2} - \frac{1}{y} \right]_4^9$$

$$= \left(\frac{2}{3} \times 27 - \frac{1}{9} \right) - \left(\frac{2}{3} \times 8 - \frac{1}{4} \right) = \frac{461}{36} \text{ units}^2$$

b)
$$\text{Vol} = \pi \int_4^9 x^2 \, dy$$

$$= \pi \int_4^9 \left(y^{1/2} + \frac{1}{y^2} \right)^2 dy$$

$$= \pi \int_4^9 \left(y + 2y^{-3/2} + \frac{1}{y^4} \right) dy$$

$$= \pi \left[\frac{y^2}{2} + \frac{2y^{-1/2}}{-1/2} + \frac{y^{-3}}{-3} \right]_4^9$$

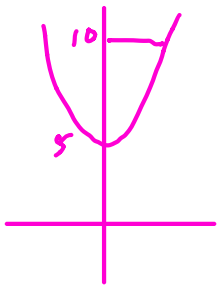
$$= \pi \left[\frac{y^2}{2} - \frac{4}{\sqrt{y}} - \frac{1}{3y^3} \right]_4^9$$

$$= \pi \left[\left(\frac{81}{2} - \frac{4}{3} - \frac{1}{2187} \right) - \left(8 - 2 - \frac{1}{192} \right) \right]$$

$$= 104.211$$

104.21 units³ to 2 d.p

Q5



$$y = 2x^2 + 5$$

$$x = 0 \Rightarrow y = 5$$

$$Vol = \int_5^{10} \pi x^2 dy$$

$$= \pi \int_5^{10} \left(\frac{y-5}{2} \right) dy$$

$$= \frac{\pi}{2} \left[\frac{y^2}{2} - 5y \right]_5^{10}$$

$$= \frac{\pi}{2} \left[(50 - 50) - \left(\frac{25}{2} - 25 \right) \right]$$

$$= \frac{\pi}{2} \times \frac{25}{2} = \frac{25\pi}{4}$$

Q7

$$y^3 + x^2 - 2y = 4$$

$$x^2 = 4 + 2y - y^3$$

$$x = \pm \sqrt{4 + 2y - y^3}$$

\therefore symmetrical about y -axis

$$\text{Vol} = \pi \int_0^2 x^2 dy$$

$$\pi \int_0^2 (4 + 2y - y^3) dy$$

$$= \pi \left[4y + y^2 - \frac{y^4}{4} \right]_0^2$$

$$= \pi \left[(8 + 4 - 4) - (0 + 0 - 0) \right]$$

$$= 8\pi$$
