

Exponential Function

$$y = e^x$$

The gradient of an exponential function is proportional to the function.

Examples $2^x, 3^x, 4^x, 5^x$

However, for number e

where $e \approx 2.718$

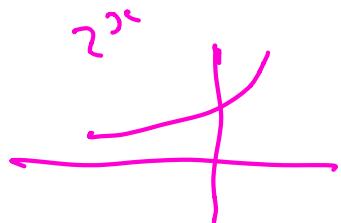
the proportional constant = 1

\therefore if $y = e^x$

$$\frac{dy}{dx} = e^x$$

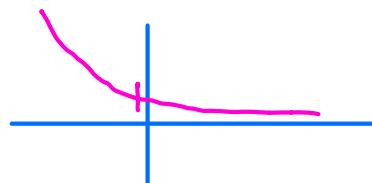
$$y = e^{kx}$$

$$\frac{dy}{dx} = ke^{kx}$$

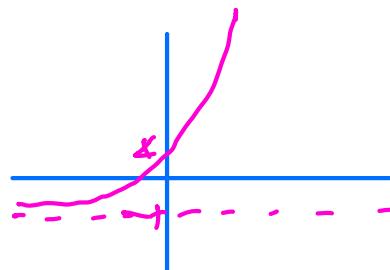


Mixed Exercise 14

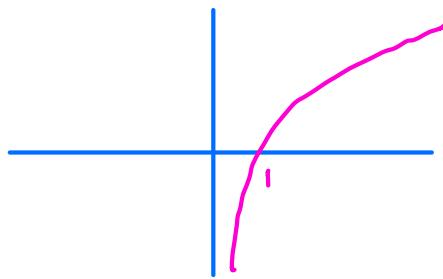
1) a) $y = 2^{-x}$



b) $y = 5e^x - 1$



c) $y = \ln x$



2 a) $\log_a(p^2q) = \log_a p^2 + \log_a q$
 $= 2 \log_a p + \log_a q$

b) $\log_a pq = 5 \Rightarrow \log_a p + \log_a q = 5 \quad ①$
 $2 \log_a p + \log_a q = 9 \quad ②$

$$\begin{array}{r} ② - ① \\ \hline 2 \log_a p + \log_a q - (\log_a p + \log_a q) = 9 - 5 \\ \hline \log_a p = 4 \\ \hline \end{array}$$
$$\Rightarrow \log_a q = 1$$

4 a) $4^x = 23$

$$\ln(4^x) = \ln 23$$

$$x \ln 4 = \ln 23$$

$$x = \frac{\ln 23}{\ln 4} = 2.26$$

7 a)

$$y = e^{-x} \quad f(x) = e^{-x} \quad \frac{d}{dx} e^{-x} = -e^{-x}$$

$$\frac{dy}{dx} = -e^{-x} \quad f'(x) = -e^{-x}$$

b) $\frac{d}{dx} e^{11x} = 11e^{11x}$

c) $\frac{d}{dx} 6e^{5x} = 30e^{5x}$

a) $P = 100 + 850e^{-\frac{1}{2}t}$

a) $t=0 \quad P = 100 + 850e^0 = £950$

b) $t=3 \quad P = 100 + 850e^{-\frac{3}{2}} = £290$

c) $200 > 100 + 850e^{-\frac{1}{2}t}$

$$100 > 850e^{-\frac{1}{2}t}$$

$$\frac{100}{850} > e^{-\frac{1}{2}t}$$

$$\ln\left(\frac{100}{850}\right) > \ln e^{-\frac{1}{2}t}$$

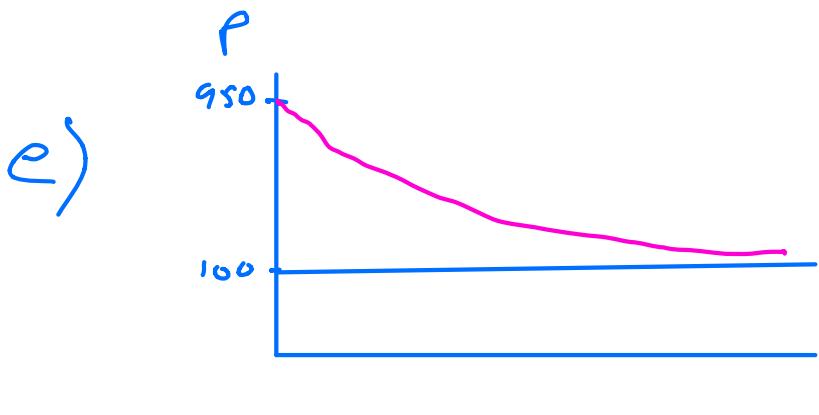
$$\ln\left(\frac{100}{850}\right) > -\frac{1}{2}t \ln e$$

$$\frac{\ln\left(\frac{100}{850}\right)}{-0.5} < t$$

$$t = 4.28 \text{ years}$$

d) As $t \rightarrow \infty$ $P \rightarrow 100 + 0$

$$P \rightarrow \$100$$



f) Maybe accurate in early years
but not accurate long term as
model suggests it will always be
worth at least \$100
