$n^{th} = ar^{n-1}$

1) a)

$3rd = ar^2 = 27$ (1)

$6^{th} = ar^5 = 8$ (2)

(2) ÷ (1) $\quad \frac{ar^5}{ar^2} = \frac{8}{27}$

$r^3 = \frac{8}{27}$

$r = \sqrt[3]{\frac{8}{27}}$

$r = \frac{2}{3}$

---

c) $S_\infty = \frac{a}{1-r}$

b) Find a

$ar^2 = 27$

$a \times \frac{4}{9} = 27$

$a = \frac{27 \times 9}{4}$

$a = \frac{243}{4}$

$S_\infty = \frac{\frac{243}{4}}{1 - \frac{2}{3}} = \frac{243}{4} \times \frac{3}{1} = \frac{729}{4}$

$= 182.25$

---

d) $S_n = \frac{a(1-r^n)}{1-r}$ $\quad S_{10} = \frac{\frac{243}{4}\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\frac{2}{3}}$

$S_{10} = 179.0895$

Difference $S_{\infty} - S_{10}$

$182.25 - 179.0895 = 3.1605$

$= 3.16$

---

11) a) Divisible by 3 and lie between 1 and 400

3, 6, 9, ... 396, 399

1st 2nd 3rd 133rd

A.P. $a = 3$, $d = 3$, $n = 133$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{133} = \frac{133}{2}(6 + 132 \times 3)$$

$$= 26,733$$

---

b) Find sum of numbers 1 to 400 not divisible by 3

= Sum of 1 to 400 − answer to part A

AP $a = 1$, $d = 1$, $n = 400$

$$S_{400} = \frac{400}{2}(2 + 399) = 80,200$$

$$\text{Solution} = 80,200 - 26,733$$

$$= 53,467$$

---

3) $u_n = 95\left(\frac{4}{5}\right)^n$

$u_1 = 76$

$u_2 = 60.8$

GP $a = 76$, $r = 0.8$ $\quad n^{\text{th}}\text{ term} = ar^{n-1}$

$u_{21} = 76 \times 0.8^{20} = 0.876$

$$\sum_{n=1}^{15} u_n = \frac{a(1 - r^n)}{1 - r}$$

$$= \frac{76(1 - 0.8^{15})}{(1 - 0.8)} = 366.62$$

$$= 367$$

$$S_\infty = \frac{a}{1-r} = \frac{76}{1-0.8} = 380$$

---

17) $a_{n+1} = \frac{1}{a_n}$ $\quad a_1 = p$

$a_1 = p \quad a_2 = \frac{1}{p} \quad a_3 = \frac{1}{\frac{1}{p}} = p$

Periodic of order 2

$$\sum_{n=1}^{1000} a_n = 500p + \frac{500}{p}$$