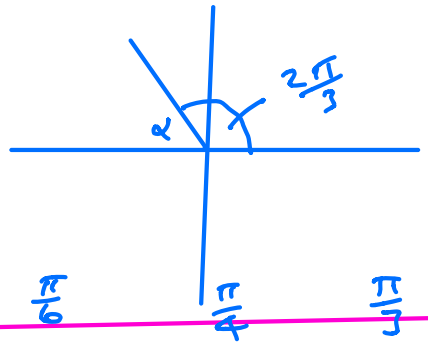
Exercise 2B Q11

$w = k + i$

$z = -4 + 5ki$

$w + z = (k-4) + (5k+1)i$

$\arg(w+z) = \frac{2\pi}{3}$



$\alpha = \frac{\pi}{3}$

$\tan\alpha = \sqrt{3}$

	0°	30° ($\frac{\pi}{6}$)	45° ($\frac{\pi}{4}$)	60° ($\frac{\pi}{3}$)	90° ($\frac{\pi}{2}$)
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\frac{\sqrt{3}}{1}$	∞

$$\frac{5k+1}{4-k} = \sqrt{3}$$

$$5k+1 = 4\sqrt{3} - \sqrt{3}k$$

$$5k + \sqrt{3}k = 4\sqrt{3} - 1$$

$$k(5+\sqrt{3}) = 4\sqrt{3} - 1$$

$$k = \frac{4\sqrt{3}-1}{5+\sqrt{3}}$$

Alternative

$z = x + yi$

$\arg z = \tan^{-1}\left(\frac{y}{x}\right)$

$$\tan^{-1}\left(\frac{y}{x}\right) = \frac{2\pi}{3}$$

$$\tan\frac{2\pi}{3} = \frac{y}{x} = \frac{5k+1}{k-4}$$

$$-\sqrt{3} = \frac{5k+1}{k-4}$$

$$-\sqrt{3}(k-4) = 5k+1$$

$$-\sqrt{3}k + 4\sqrt{3} = 5k+1$$

$$4\sqrt{3}-1 = 5k + \sqrt{3}k$$

$$4\sqrt{3}-1 = k(5+\sqrt{3})$$

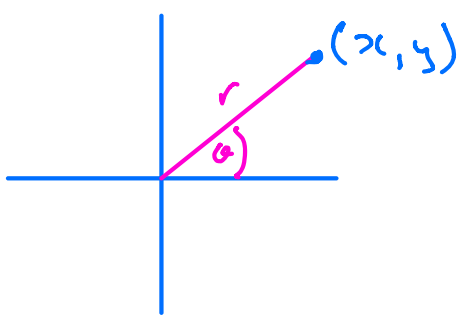
$$\frac{4\sqrt{3}-1}{5+\sqrt{3}} = k$$

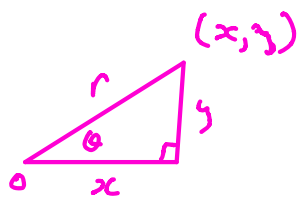
Modulus -Argument form of complex numbers

For a complex number z with $|z| = r$
and $\arg z = \theta$

$$z = r(\cos\theta + i\sin\theta)$$

If $z = x + iy$ then $x = r\cos\theta$
$y = r\sin\theta$





$$\cos\theta = \frac{x}{r}$$

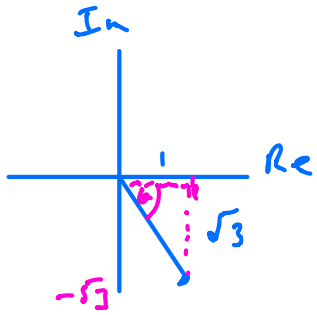
$$\underline{x = r\cos\theta}$$

$$\sin\theta = \frac{y}{r}$$

$$\underline{y = r\sin\theta}$$

Exercise 2C

1) d)



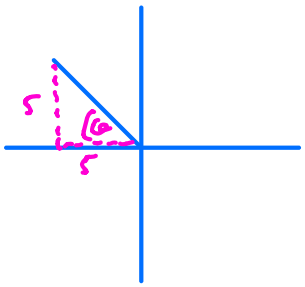
$$\tan\theta = \frac{\sqrt{3}}{1} \Rightarrow \theta = \frac{\pi}{3}$$

$$\arg z = -\frac{\pi}{3}$$

$$r = \sqrt{\sqrt{3}^2 + (-1)^2} = 2$$

$$z = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

1) h)



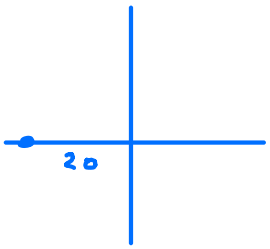
$$\theta = \tan^{-1}\frac{5}{5} = \frac{\pi}{4}$$

$$\arg z = \frac{3\pi}{4}$$

$$r = \sqrt{(-5)^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

$$z = 5\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

1 f)



$$z = 20(\cos\pi + i\sin\pi)$$

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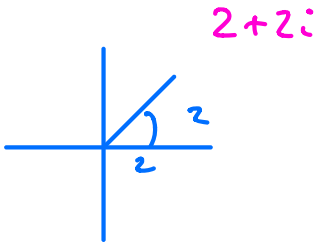
1 a)

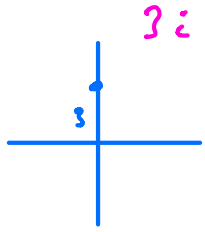


$$|z| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\theta = \frac{\pi}{4}$$

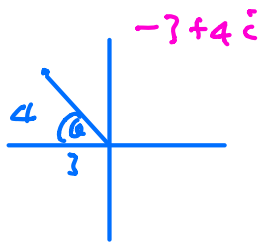
$$z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

1b)



$$z = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

1c)



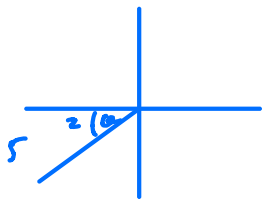
$$\theta = \tan^{-1}\frac{4}{3}$$

$$\arg z = \pi - \tan^{-1}\frac{4}{3} \approx 2.21$$

$$|z| = \sqrt{(-3)^2+4^2} = 5$$

$$z = 5(\cos 2.21 + i\sin 2.21)$$

1e) $-2-5i$



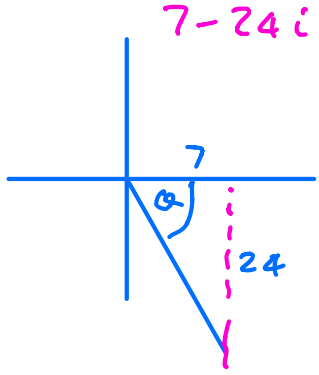
$$\theta = \tan^{-1}\frac{5}{2}$$

$$\arg z = -\pi + \tan^{-1}\frac{5}{2} = -1.95$$

$$|z| = \sqrt{(-2)^2+(-5)^2} = \sqrt{29}$$

$$z = \sqrt{29}\left(\cos(-1.95) + i\sin(-1.95)\right)$$

1g)



$$\theta = \tan^{-1}\frac{24}{7}$$

$$\arg z = -\tan^{-1}\frac{24}{7} = -1.29$$

$$|z| = \sqrt{7^2+(-24)^2} = 25$$

$$z = \sqrt{29}\left(\cos(-1.29) + i\sin(-1.29)\right)$$