

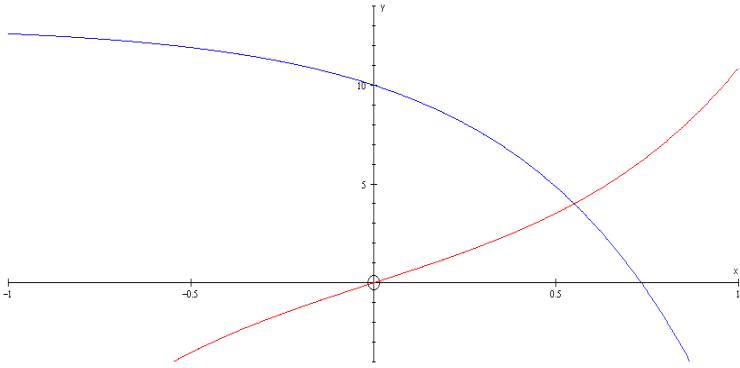
June 2009
6669 Further Pure Mathematics FP3 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{7}{\cosh x} - \frac{\sinh x}{\cosh x} = 5 \Rightarrow \frac{14}{e^x + e^{-x}} - \frac{(e^x - e^{-x})}{e^x + e^{-x}} = 5$ $\therefore 14 - (e^x - e^{-x}) = 5(e^x + e^{-x}) \Rightarrow 6e^x - 14 + 4e^{-x} = 0$ $\therefore 3e^{2x} - 7e^x + 2 = 0 \Rightarrow (3e^x - 1)(e^x - 2) = 0$ $\therefore e^x = \frac{1}{3} \text{ or } 2$ $x = \ln(\frac{1}{3}) \text{ or } \ln 2$	M1 A1 M1 A1 B1ft [5]
Alternative (i)	Write $7 - \sinh x = 5 \cosh x$, then use exponential substitution $7 - \frac{1}{2}(e^x - e^{-x}) = \frac{5}{2}(e^x + e^{-x})$ Then proceed as method above.	M1
Alternative (ii)	$(7 \operatorname{sech} x - 5)^2 = \tanh^2 x = 1 - \operatorname{sech}^2 x$ $50 \operatorname{sech}^2 x - 70 \operatorname{sech} x + 24 = 0$ $2(5 \operatorname{sech} x - 3)(5 \operatorname{sech} x - 4) = 0$ $\operatorname{sech} x = \frac{3}{5} \text{ or } \operatorname{sech} x = \frac{4}{5}$ $x = \ln(\frac{1}{3}) \text{ or } \ln 2$	M1 A1 M1 A1 B1ft

Question Number	Scheme	Marks
Q4 (a)	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \times \frac{1}{\sqrt{1+(\sqrt{x})^2}}$ $\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \left(= \frac{1}{2\sqrt{x(1+x)}} \right)$	B1, M1
(b)	$\therefore \int_{\frac{1}{4}}^4 \frac{1}{\sqrt{x(x+1)}} dx = [2\operatorname{arsinh} \sqrt{x}]_{\frac{1}{4}}^4$ $= [2\operatorname{arsinh} 2 - 2\operatorname{arsinh}(\frac{1}{2})]$ $= [2\ln(2+\sqrt{5})] - [2\ln(\frac{1}{2} + \sqrt{\frac{5}{4}})]$ $2\ln \frac{(2+\sqrt{5})}{(\frac{1}{2} + \sqrt{\frac{5}{4}})} = 2\ln \frac{2(2+\sqrt{5})}{(1+\sqrt{5})} = 2\ln \frac{2(\sqrt{5}+2)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = 2\ln \frac{(3+\sqrt{5})}{2}$ $= \ln \frac{(3+\sqrt{5})(3+\sqrt{5})}{4} = \ln \frac{14+6\sqrt{5}}{4} = \ln \left(\frac{7}{2} + \frac{3\sqrt{5}}{2} \right)$	A1 (3) M1 M1 M1 M1 A1 A1 (6) [9]
Alternative (i) for part (a)	<p>Use $\sinh y = \sqrt{x}$ and state $\cosh y \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$</p> $\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+\sinh^2 y}} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+(\sqrt{x})^2}}$ $\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \left(= \frac{1}{2\sqrt{x(1+x)}} \right)$	B1 M1 A1 (3)
Alternative (i) for part (b)	<p>Use $x = \tan^2 \theta$, $\frac{dx}{d\theta} = 2\tan \theta \sec^2 \theta$ to give $2 \int \sec \theta d\theta = [2\ln(\sec \theta + \tan \theta)]$</p> $= [2\ln(\sec \theta + \tan \theta)]_{\tan \theta = \frac{1}{2}}^{\tan \theta = 2}$ <p>i.e. use of limits</p> <p>then proceed as before from line 3 of scheme</p>	M1 M1
Alternative (ii) for part (b)	<p>Use $\int \frac{1}{\sqrt{[(x+\frac{1}{2})^2 - \frac{1}{4}]}} dx = \operatorname{arcosh} \frac{x+\frac{1}{2}}{\frac{1}{2}}$</p> $= [\operatorname{arcosh} 9 - \operatorname{arcosh}(\frac{3}{2})]$ $= [\ln(9+\sqrt{80})] - [\ln(\frac{3}{2} + \frac{1}{2}\sqrt{5})]$ $= \ln \frac{(9+\sqrt{80})}{(\frac{3}{2} + \frac{1}{2}\sqrt{5})} = \ln \frac{2(9+\sqrt{80})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})},$ $= \ln \frac{2(9+4\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \ln \left(\frac{7}{2} + \frac{3\sqrt{5}}{2} \right)$	M1 M1 M1 M1 M1 A1 A1 (6) [9]

Question Number	Scheme	Marks
3(a)	$\text{rhs} = 1 + 2 \sinh^2 x = 1 + 2 \left(\frac{e^x - e^{-x}}{2} \right)^2$ $= \frac{2 + e^{2x} - 2 + e^{-2x}}{2}$ $= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x = \text{lhs} \quad *$	M1 M1 A1 (3)
(b)	$1 + 2 \sinh^2 x - 3 \sinh x = 15$ $2 \sinh^2 x - 3 \sinh x - 14 = 0$ $(\sinh x + 2)(2 \sinh x - 7) = 0$ $\sinh x = -2, \frac{7}{2}$ $x = \ln \left(-2 + \sqrt{(-2)^2 + 1} \right) = \ln \left(-2 + \sqrt{5} \right)$ $x = \ln \left(\frac{7}{2} + \sqrt{\left(\frac{7}{2} \right)^2 + 1} \right) = \ln \left(\frac{7 + \sqrt{53}}{2} \right)$	M1 M1 A1 M1 A1 (5)
		8

Question Number	Scheme	Marks
5(a)	$\frac{dy}{dx} = 2 \operatorname{ar cosh}(3x) \times \frac{3}{\sqrt{9x^2 - 1}}$ $\sqrt{9x^2 - 1} \frac{dy}{dx} = 6 \operatorname{ar cosh}(3x)$ $(9x^2 - 1) \left(\frac{dy}{dx} \right)^2 = 36 (\operatorname{ar cosh}(3x))^2$ $(9x^2 - 1) \left(\frac{dy}{dx} \right)^2 = 36y \quad *$	M1A1A1 dM1 A1 (5)
(b)	$\left\{ 18x \left(\frac{dy}{dx} \right)^2 + (9x^2 - 1) \times 2 \frac{dy}{dx} \times \frac{d^2 y}{dx^2} \right\} = 36 \frac{dy}{dx}$ $(9x^2 - 1) \frac{d^2 y}{dx^2} + 9x \frac{dy}{dx} = 18 \quad *$	M1 {A1} A1 A1 (4) 9

Question Number	Scheme	Marks
5. (a)	 <p>Graph of $y = 3\sinh 2x$</p> <p>Shape of $-e^{2x}$ graph</p> <p>Asymptote: $y = 13$</p> <p>Value 10 on y axis and value 0.7 or $\frac{1}{2} \ln\left(\frac{13}{3}\right)$ on x axis</p>	B1 B1 B1 B1 B1 (4)
(b)	<p>Use definition $\frac{3}{2}(e^{2x} - e^{-2x}) = 13 - 3e^{2x} \rightarrow 9e^{4x} - 26e^{2x} - 3 = 0$ to form quadratic</p> $\therefore e^{2x} = -\frac{1}{9} \text{ or } 3$ $\therefore x = \frac{1}{2} \ln(3)$	M1 A1 DM1 A1 B1 (5) 9
	<p>Notes:</p> <p>(a) 1B1 $y = 3\sinh 2x$ first and third quadrant.</p> <p>2B1 Shape of $y = -e^{2x}$ correct intersects on positive axes.</p> <p>3B1 Equation of asymptote, $y = 13$, given. Penlise 'extra' asymptotes here</p> <p>4B1 Intercepts correct both</p> <p>(b) 1M1 Getting a three terms quadratic in e^{2x}</p> <p>1A1 Correct three term quadratic</p> <p>2DM1 Solving for e^{2x}</p> <p>2A1 CAO for e^{2x} condone omission of negative value.</p> <p>B1 CAO one answer only</p>	

Question Number	Scheme	Marks
5. (a)	$\text{arsinh } 2x, +x \frac{2}{\sqrt{1+4x^2}}$	M1A1, A1 (3)
(b)	$\begin{aligned} \therefore \int_0^{\sqrt{2}} \text{arsinh} 2x \, dx &= [x \text{arsinh} 2x]_0^{\sqrt{2}} - \int_0^{\sqrt{2}} \frac{2x}{\sqrt{1+4x^2}} \, dx \\ &= [x \text{arsinh} 2x]_0^{\sqrt{2}} - \left[\frac{1}{2} (1+4x^2)^{\frac{1}{2}} \right]_0^{\sqrt{2}} \\ &= \sqrt{2} \text{arsinh} 2\sqrt{2} - [\frac{3}{2} - \frac{1}{2}] \\ &= \sqrt{2} \ln(3+2\sqrt{2}) - 1 \end{aligned}$	1M1 1A1ft 2M1 2A1 3DM1 4M1 3A1 (7) (10 marks)

Notes

a1M1: Differentiating getting an arsinh term **and** a term of the form $\frac{px}{\sqrt{1\pm qx^2}}$

a1A1: cao arsinh $2x$

a2A1: cao + $\frac{2x}{\sqrt{1+4x^2}}$

b1M1: rearranging their answer to (a). **OR** setting up parts

b1A1: ft from their (a) **OR** setting up parts correctly

b2M1: Integrating getting an arsinh or arcosh term **and** a $(1\pm ax^2)^{\frac{1}{2}}$ term o.e..

b2A1: cao

b3DM1: depends on previous M, correct use of $\sqrt{2}$ and 0 as limits.

b4M1: converting to log form.

b3A1: cao depends on all previous M marks.

Question Number	Scheme	Marks
7(a)	$f(x) = 5\cosh x - 4\sinh x = 5 \times \frac{1}{2}(e^x + e^{-x}) - 4 \times \frac{1}{2}(e^x - e^{-x})$ $= \frac{1}{2}(e^x + 9e^{-x}) \quad *$	M1 A1cso (2)
(b)	$\frac{1}{2}(e^x + 9e^{-x}) = 5 \Rightarrow e^{2x} - 10e^x + 9 = 0$ <p>So $e^x = 9$ or 1 and $x = \ln 9$ or 0</p>	M1 A1 M1 A1 (4)
(c)	<p>Integral may be written $\int \frac{2e^x}{e^{2x} + 9} dx$</p> <p>This is $\frac{2}{3} \arctan\left(\frac{e^x}{3}\right)$</p> <p>Uses limits to give $\left[\frac{2}{3} \arctan 1 - \frac{2}{3} \arctan\left(\frac{1}{\sqrt{3}}\right) \right] = \left[\frac{2}{3} \times \frac{\pi}{4} - \frac{2}{3} \times \frac{\pi}{6} \right] = \frac{\pi}{18} *$</p>	B1 M1 A1 DM1 A1cso (5) (11 marks)

Notes

a1M1: Replacing both coshx and sinh x by terms in e^x and e^{-x} condone sign errors here.

a1A1: cso (answer given)

b1M1: Getting a three term quadratic in e^x

b1A1: cao

b2M1: solving to $x =$

b2A1: cao need $\ln 9$ (o.e) and 0 (not $\ln 1$)

c1B1: cao getting into suitable form, may substitute first.

c1M1: Integrating to give term in arctan

c1A1: cao

c2M1: Depends on previous M mark. Correct use of $\ln 3$ and $\frac{1}{2} \ln 3$ as limits.

c2A1: cso must see them subtracting two terms in π .