Basic Differentiation 2 Solutions

5. $$f(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found.

(3)

(b) Find $f'(x)$.

(3)

(c) Evaluate $f'(9)$.

(2)

(Total 8 marks)

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a) $$f(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}} = \frac{9 - 24\sqrt{x} + 16x}{\sqrt{x}}$$

$$f(x) = \frac{9}{\sqrt{x}} - 24 + 16\sqrt{x}$$

$$f(x) = 9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$$

b) $$f'(x) = -\frac{9}{2}x^{-\frac{3}{2}} + 8x^{-\frac{1}{2}}$$

c) $f'(9) = -\frac{9}{2}(9)^{-\frac{3}{2}} + 8(9)^{-\frac{1}{2}}$

$f'(9) = -\frac{9}{2}\left(\frac{1}{9}\right)^{3/2} + 8\left(\frac{1}{9}\right)^{\frac{1}{2}}$

$f'(9) = -\frac{9}{2}\left(\frac{1}{27}\right) + 8\left(\frac{1}{3}\right)$

$f'(9) = -\frac{1}{6} + \frac{8}{3}$

$f'(9) = 2\frac{1}{2}$

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6. Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^2 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)

(Total 6 marks)

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a) $\dfrac{2x^2 - x^{3/2}}{\sqrt{x}} = 2x^{3/2} - x$

$\Rightarrow p = \frac{3}{2}, \; q = 1$

b) $$y = 5x^2 - 3 + \frac{2x^2 - x^{3/2}}{\sqrt{x}}$$

$$y = 5x^2 - 3 + 2x^{3/2} - x$$

$$\frac{dy}{dx} = 10x + 3x^{\frac{1}{2}} - 1$$

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7. $$f(x) = 3x + x^3,\ x > 0.$$

(a) Differentiate to find $f'(x)$.

(2)

Given that $f'(x) = 15$,

(b) find the value of x.

(3)
(Total 5 marks)

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a) $$f(x) = 3x + x^3$$

$$f'(x) = 3 + 3x^2$$

b) $$15 = 3 + 3x^2$$

$$12 = 3x^2$$

$$4 = x^2$$

$$x = 2 \quad \text{since } x > 0$$

8. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k,

(4)

(c) the value of the y-coordinate of A.

(2)

(Total 8 marks)

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a)

$$y = kx^3 - x^2 + x - 5$$

$$\frac{dy}{dx} = 3kx^2 - 2x + 1$$

b) line is $2y - 7x + 1 = 0$

$$2y = 7x - 1$$

$$y = \frac{7}{2}x - \frac{1}{2}$$

so gradient of tgt $= \frac{7}{2}$

Given that x-coord $= -\frac{1}{2}$

Sub in $\frac{dy}{dx}$

$$\frac{7}{2} = 3k\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1$$

$$\frac{7}{2} = \frac{3}{4}k + 1 + 1$$

$$\frac{3}{2} = \frac{3}{4}k$$

$$\frac{3}{2} \times \frac{4}{3} = k$$

$$\underline{k = 2}$$

c)

$$y = 2x^3 - x^2 + x - 5$$

when $x = -\frac{1}{2}$

$$y = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 5$$

$$y = -\frac{1}{4} - \frac{1}{4} - \frac{1}{2} - 5$$

$$\underline{y = -6}$$

9. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)

(Total 6 marks)

a) $\dfrac{2\sqrt{x}+3}{x} = \dfrac{2}{\sqrt{x}} + \dfrac{3}{x} = 2x^{-\frac{1}{2}} + 3x^{-1}$

b) $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$

$y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}$

$\dfrac{dy}{dx} = 5 - x^{-3/2} - 3x^{-2}$