Algebraic Proof 2

Algebraic Proofs GCSE Higher Tier A/A*Grades KS4 with Answers/Solutions

1. Prove that $(n+4)^2 - (3n+4) = (n+1)(n+4) + 8$
2. Prove that $(n+4)^2 - (3n+4) = (n+2)(n+3) + 6$
3. Prove that $(n+3)^2 - (3n+5) = (n+1)(n+2) + 2$
4. Prove that $(n-5)^2 - (2n-1) = (n-3)(n-9) - 1$
5. Prove that $(n-3)^2 - (n-5) = (n-3)(n-4) + 2$
6. Prove that $\frac{1}{2}(n+1)(n+2) - \frac{1}{2}n(n+1) = n+1$
7. Prove that $\frac{1}{4}(2n+1)(n+4) - \frac{1}{4}n(2n+1) = 2n+1$

8. Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of 6 for all positive integer values of n.
9. Prove that $(4n+1)^2 - (4n-1)^2$ is a multiple of 8 for all positive integer values of n.
10. Prove that $(5n+1)^2 - (5n-1)^2$ is a multiple of 5 for all positive integer values of n.

11. Prove that $(2n+1)^2 - (2n-1)^2$ is a multiple of 8 for all positive integer values of n.
12. Prove that $(5n+1)^2 - (5n-1)^2$ is a multiple of 4 for all positive integer values of n.
13. Prove that $(2n+1)^2 - (2n-1)^2 - 10$ is not a multiple of 8 for all positive integer values of n.
14. Prove that $(2n+1)^2 - (2n-1)^2 - 2$ is not a multiple of 4 for all positive integer values of n.
15. Prove that $(n+1)^2 - (n-1)^2 + 1$ is always odd for all positive integer values of n.
16. Prove that $(n+1)^2 - (n-1)^2 + 4$ is always even for all positive integer values of n.

7. Prove that $\frac{1}{4}(2n+1)(n+4) - \frac{1}{4}n(2n+1) = 2n+1$

$$\frac{1}{4}\left[2n^2 + n + 8n + 4\right] - \frac{1}{2}n^2 - \frac{1}{4}n$$

$$\cancel{\frac{1}{2}n^2} + \frac{9}{4}n + 1 - \cancel{\frac{1}{2}n^2} - \frac{1}{4}n = 2n + 1$$

11. Prove that $(2n+1)^2 - (2n-1)^2$ is a multiple of 8 for all positive integer values of n.

$$(2n+1)^2 - (2n-1)^2$$

$$= 4n^2 + 1 + 4n - \left[4n^2 + 1 - 4n\right]$$

$$= \cancel{4n^2} + \cancel{1} + 4n - \cancel{4n^2} - \cancel{1} + 4n = 8n$$

a multiple of 8

13. Prove that $(2n+1)^2 - (2n-1)^2 - 10$ is not a multiple of 8 for all positive integer values of n.

$$(2n+1)^2 - (2n-1)^2 - 10$$

$$= 4n^2 + 1 + 4n - \left[4n^2 + 1 - 4n\right] - 10$$

$$= \cancel{4n^2} + \cancel{1} + 4n - \cancel{4n^2} - \cancel{1} + 4n - 10$$

$$= 8n - 10$$

$$= 8n - 8 - 2$$

$$= 8(n-1) - 2$$ not a multiple of 8

6. Prove that $\frac{1}{2}(n+1)(n+2) - \frac{1}{2}n(n+1) = n+1$

$\frac{1}{2}\left[n^2+n+2n+2\right] - \frac{1}{2}n^2 - \frac{1}{2}n$

$= \cancel{\frac{1}{2}n^2} + \frac{3}{2}n + 1 - \cancel{\frac{1}{2}n^2} - \frac{1}{2}n = n+1$

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10. Prove that $(5n+1)^2 - (5n-1)^2$ is a multiple of 5 for all positive integer values of n.

$25n^2 + 1 + 10n - \left[25n^2 + 1 - 10n\right]$

$= \cancel{25n^2} \cancel{+1} + 10n \cancel{-25n^2} \cancel{-1} + 10n$

$= 20n$

$= 5(4n)$ a multiple of 5

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12. Prove that $(5n+1)^2 - (5n-1)^2$ is a multiple of 4 for all positive integer values of n.

Same as Q10

$= 20n$

$= 4(5n)$ a multiple of 4

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14. Prove that $(2n+1)^2 - (2n-1)^2 - 2$ is not a multiple of 4 for all positive integer values of n.

$= 4n^2 + 1 + 4n - \left[4n^2 + 1 - 4n\right] - 2$

$= \cancel{4n^2} + \cancel{1} + 4n - \cancel{4n^2} - \cancel{1} + 4n - 2$

$= 8n - 2$

$= 4(2n) - 2$ not a multiple of 4

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16. Prove that $(n+1)^2 - (n-1)^2 + 4$ is always even for all positive integer values of n.

$n^2 + 1 + 2n - [n^2 + 1 - 2n] + 4$

$= \cancel{n^2} + \cancel{1} + 2n - \cancel{n^2} - \cancel{1} + 2n + 4$

$= 4n + 4$

$= 2(2n+2)$ which is even

In all the questions below, n is a positive integer.

17. If $2n$ is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

18. If $(2n + 1)$ is always odd for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive odd numbers cannot be a multiple of 4.

19. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

20. Prove algebraically that the difference between the squares of any two consecutive even numbers is always a multiple of 4.

21. Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

22. Prove algebraically that the difference between the squares of any two consecutive numbers is always an odd number

23. Prove algebraically that the sum of the squares of any three consecutive even numbers always a multiple of 4.

24. Prove algebraically that the sum of the squares of any three consecutive odd numbers always leaves a remainder of 11 when divided by 12.

17. If $2n$ is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

Let two consecutive even numbers be

$2n$ and $2n+2$

$$(2n)^2 + (2n+2)^2$$
$$= 4n^2 + 4n^2 + 4 + 8n$$
$$= 8n^2 + 8n + 4$$
$$= 4(2n^2 + 2n + 1) \quad \text{a multiple of 4}$$

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19. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

Let numbers be n and $n+1$

$$n^2 + (n+1)^2$$
$$= n^2 + n^2 + 1 + 2n$$
$$= 2n^2 + 2n + 1$$
$$= 2(n^2 + n) + 1$$
$$= 2n(n+1) + 1$$

Either n or $(n+1)$ is even and so has a factor of 2. Therefore $2 \times n(n+1)$ has a factor of 4. The first term is therefore a multiple of 4 and $+1$ is the remainder

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21. Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

Let consecutive odd numbers be $2n+1$, $2n+3$

$$(2n+3)^2 - (2n+1)^2$$

$$4n^2 + 9 + 12n - [4n^2 + 1 + 4n]$$

$$= \cancel{4n^2} + 9 + 12n - \cancel{4n^2} - 1 - 4n$$

$$= 8n + 8$$

$$= 8(n+1) \quad \text{a multiple of } 8$$

23. Prove algebraically that the sum of the squares of any three consecutive even numbers always a multiple of 4.

Numbers $2n$, $2n+2$, $2n+4$

$$(2n)^2 + (2n+2)^2 + (2n+4)^2$$

$$= 4n^2 + 4n^2 + 4 + 8n + 4n^2 + 16 + 16n$$

$$= 12n^2 + 24n + 20$$

$$= 4(3n^2 + 6n + 5) \quad \text{a multiple of } 4$$

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