Jan 05

1 The position vector, $\mathbf{r}$, of a particle of mass 4 kg at time t is given by

$$\mathbf{r} = t^2\mathbf{i} + (5t - 2t^2)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors, lengths are in metres and time is in seconds.

(i) Find an expression for the acceleration of the particle. [4]

The particle is subject to a force $\mathbf{F}$ and a force $12\mathbf{j}$ N.

(ii) Find $\mathbf{F}$. [3]

Jun 05

5 The position vector of a particle at time t is given by

$$\mathbf{r} = \tfrac{1}{2}t\mathbf{i} + (t^2 - 1)\mathbf{j},$$

referred to an origin O where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors in the directions of the cartesian axes Ox and Oy respectively.

(i) Write down the value of t for which the x-coordinate of the position of the particle is 2. Find the y-coordinate at this time. [2]

(ii) Show that the cartesian equation of the path of the particle is $y = 4x^2 - 1$. [2]

Jun 06

4 Fig. 4 shows the unit vectors $\mathbf{i}$ and $\mathbf{j}$ in the directions of the cartesian axes Ox and Oy, respectively. O is the origin of the axes and of position vectors.

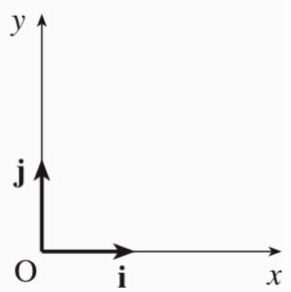

Fig. 4

The position vector of a particle is given by $\mathbf{r} = 3t\mathbf{i} + (18t^2 - 1)\mathbf{j}$ for $t \geqslant 0$, where t is time.

(i) Show that the path of the particle cuts the x-axis just once. [2]

(ii) Find an expression for the velocity of the particle at time t.

Deduce that the particle never travels in the $\mathbf{j}$ direction. [3]

(iii) Find the cartesian equation of the path of the particle, simplifying your answer. [3]

Jan 07

6 The velocity of a model boat, $\mathbf{v}$ m s^{-1}, is given by

$$\mathbf{v} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + t \begin{pmatrix} 6 \\ -8 \end{pmatrix},$$

where t is the time in seconds and the vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are east and north respectively.

(i) Show that when $t = 2.5$ the boat is travelling south-east (i.e. on a bearing of 135°). Calculate its speed at this time. [3]

The boat is at a point O when $t = 0$.

(ii) Calculate the bearing of the boat from O when $t = 2.5$. [4]

Jun 07

6 A rock of mass 8 kg is acted on by just the two forces $-80\mathbf{k}$ N and $(-\mathbf{i} + 16\mathbf{j} + 72\mathbf{k})$ N, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane and $\mathbf{k}$ is a unit vector vertically upward.

(i) Show that the acceleration of the rock is $\left(-\frac{1}{8}\mathbf{i} + 2\mathbf{j} - \mathbf{k}\right)$ m s^{-2}. [2]

The rock passes through the origin of position vectors, O, with velocity $(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$ m s^{-1} and 4 seconds later passes through the point A.

(ii) Find the position vector of A. [3]

(iii) Find the distance OA. [1]

(iv) Find the angle that OA makes with the horizontal. [2]

Jan 08

2 The force acting on a particle of mass 1.5 kg is given by the vector $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$ N.

(i) Give the acceleration of the particle as a vector. [2]

(ii) Calculate the angle that the acceleration makes with the direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. [2]

(iii) At a certain point of its motion, the particle has a velocity of $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ m s^{-1}. Calculate the displacement of the particle over the next two seconds. [3]

Jun 08

3 An object of mass 5 kg has a constant acceleration of $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m s^{-2} for $0 \leqslant t \leqslant 4$, where t is the time in seconds.

(i) Calculate the force acting on the object. [2]

When $t = 0$, the object has position vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ m and velocity $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ m s^{-1}.

(ii) Find the position vector of the object when $t = 4$. [3]

Jan 09

8 A toy boat moves in a horizontal plane with position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors east and north respectively. The origin of the position vectors is at O. The displacements x and y are in metres.

First consider only the motion of the boat parallel to the x-axis. For this motion

$$x = 8t - 2t^2.$$

The velocity of the boat in the x-direction is v_x m s^{-1}.

(i) Find an expression in terms of t for v_x and determine when the boat instantaneously has zero speed in the x-direction. **[3]**

Now consider only the motion of the boat parallel to the y-axis. For this motion

$$v_y = (t - 2)(3t - 2),$$

where v_y m s^{-1} is the velocity of the boat in the y-direction at time t seconds.

(ii) Given that $y = 3$ when $t = 1$, use integration to show that $y = t^3 - 4t^2 + 4t + 2$. **[4]**

The position vector of the boat is given in terms of t by $\mathbf{r} = (8t - 2t^2)\mathbf{i} + (t^3 - 4t^2 + 4t + 2)\mathbf{j}$.

(iii) Find the time(s) when the boat is due north of O and also the distance of the boat from O at any such times. **[4]**

(iv) Find the time(s) when the boat is instantaneously at rest. Find the distance of the boat from O at any such times. **[5]**

(v) Plot a graph of the path of the boat for $0 \leqslant t \leqslant 2$. **[3]**

Jun 09

5 The position vector of a toy boat of mass 1.5 kg is modelled as $\mathbf{r} = (2 + t)\mathbf{i} + (3t - t^2)\mathbf{j}$ where lengths are in metres, t is the time in seconds, $\mathbf{i}$ and $\mathbf{j}$ are horizontal, perpendicular unit vectors and the origin is O.

(i) Find the velocity of the boat when $t = 4$. **[3]**

(ii) Find the acceleration of the boat and the horizontal force acting on the boat. **[3]**

(iii) Find the cartesian equation of the path of the boat referred to x- and y-axes in the directions of $\mathbf{i}$ and $\mathbf{j}$, respectively, with origin O. You are not required to simplify your answer. **[2]**

Jan 10

2 A particle of mass 5 kg has constant acceleration. Initially, the particle is at $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m with velocity $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ m s^{-1}; after 4 seconds the particle has velocity $\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ m s^{-1}.

(i) Calculate the acceleration of the particle. **[2]**

(ii) Calculate the position of the particle at the end of the 4 seconds. **[3]**

(iii) Calculate the force acting on the particle. **[2]**