Review-Projectiles and Moments

6.

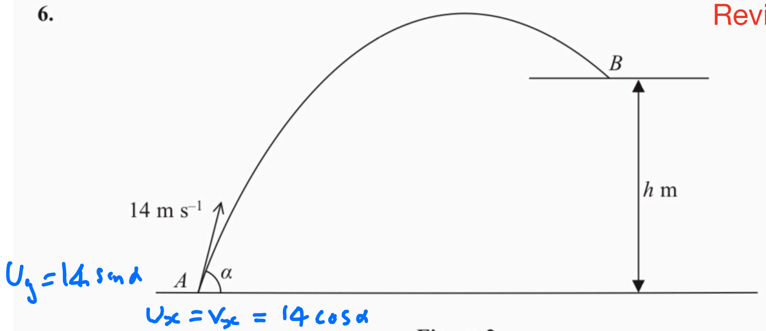

Figure 2

A small ball is projected with speed 14 m s^{-1} from a point A on horizontal ground. The angle of projection is α above the horizontal. A horizontal platform is at height h metres above the ground. The ball moves freely under gravity until it hits the platform at the point B, as shown in Figure 2. The speed of the ball immediately before it hits the platform at B is 10 m s^{-1}.

(a) Find the value of h.

(4)

Given that $\sin\alpha = 0.85$,

(b) find the horizontal distance from A to B.

(8)

a) $\updownarrow$ $\quad V_y^2 = u_y^2 - 19.6y$

Speed 10 ms^{-1} at B $\Rightarrow$ $V_x^2 + V_y^2 = 100$

$$(14\cos\alpha)^2 + (14\sin\alpha)^2 - 19.6h = 100$$

$$196 - 19.6h = 100$$

$$196 - 100 = 19.6h$$

$$96 = 19.6h$$

$$\frac{96}{19.6} = h$$

$$h = 4.90\text{m}$$

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b) Find time to reach B

$$V_y = u_y t - 4.9t^2$$

$$4.90 = 14 \sin\alpha t - 4.9t^2$$

$$4.90 = 14 \times 0.85t - 4.9t^2$$

$$4.9t^2 - 11.9t + 4.90 = 0$$

By calc $\underline{t = 1.903}$, $t = 0.525$ s

From graph $t = 1.903$

Horizontal distance $x = U_x t$

$$x = 14 \cos\alpha \times 1.903$$

$$x = 14\sqrt{1 - .85^2} \times 1.903$$

$$x = 14.0 \text{ m}$$

7.

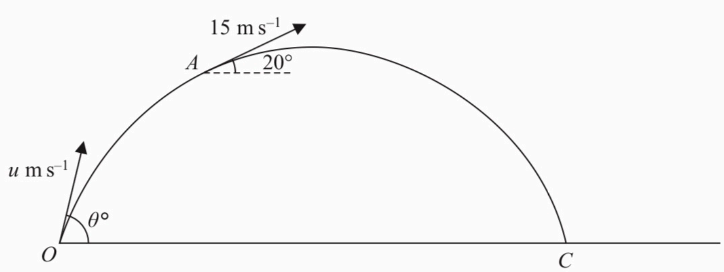


Figure 3

At time $t = 0$, a particle is projected from a fixed point O on horizontal ground with speed u m s^{-1} at an angle $\theta°$ to the horizontal. The particle moves freely under gravity and passes through the point A when $t = 4$ s. As it passes through A, the particle is moving upwards at 20° to the horizontal with speed 15 m s^{-1}, as shown in Figure 3.

(a) Find the value of u and the value of θ.
(7)

At the point B on its path the particle is moving downwards at 20° to the horizontal with speed 15 m s^{-1}.

(b) Find the time taken for the particle to move from A to B.
(2)

The particle reaches the ground at the point C.

(c) Find the distance OC.
(3)

a) $\updownarrow$ $U_y = U \sin\theta$

$\leftrightarrow$ $U_x = V_x = U\cos\theta$

At A

$V_x = 15\cos 20^\circ = 14.095\ ms^{-1}$

$\therefore \quad \underline{U\cos\theta = 14.095}$ (1)

Vertically at A

$V_y = 15\sin 20^\circ$

But $V_y = U_y - 9.8t$ At A $t=4$

$\Rightarrow \quad 15\sin 20 = U\sin\theta - 9.8\times 4$

$15\sin 20^\circ + 39.2 = U\sin\theta$

$\underline{U\sin\theta = 44.330}$ (2)

(2) $\div$ (1) $\quad \dfrac{U\sin\theta}{U\cos\theta} = \tan\theta = \dfrac{44.330}{14.095}$

$\theta = \tan^{-1}\left(\dfrac{44.330}{14.095}\right)$

$\underline{\theta = 72.361^\circ}$

Sub in (2) $\quad U = \dfrac{44.330}{\sin 72.361^\circ}$

$\underline{U = 46.5\ ms^{-1}}$

$\underline{\theta = 72.4^\circ}$

b) $V_y = U_y - 9.8t$

$V_y = U\sin\theta - 9.8t$

$V_y = 44.330 - 9.8t$

$15\sin 20° = 44.330 - 9.8t$

$9.8t = 44.330 - 15\sin 20$

$t = \dfrac{44.330 - 15\sin 20}{9.8} = 4s$

On way down

$-15\sin 20 = 44.330 - 9.8t_2$

$9.8t_2 = 44.330 + 15\sin 20$

$t_2 = \dfrac{44.330 + 15\sin 20}{9.8} = 5.047s$

Time taken $= 5.047 - 4$

$= \underline{1.05 \text{ s}}$

c) Reaches ground when $y = 0$

$y = U_y t - 4.9t^2$

$0 = 44.33t - 4.9t^2$

$0 = t(44.33 - 4.9t)$

lands when $44.33 - 4.9t = 0$

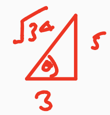
$t = \dfrac{44.33}{4.9} = 9.047$

$OC = U_x t = 46.5\cos 72.4° \times 9.047$

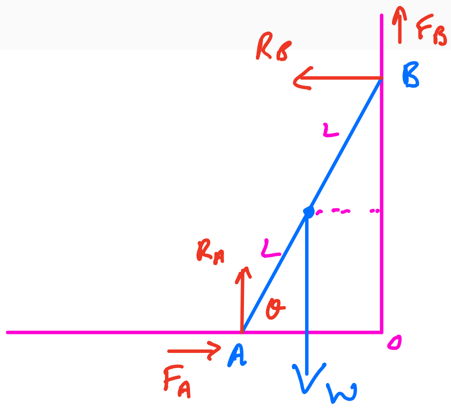
$OC = 127.2\text{ m} = \underline{127\text{ m}}$

4. A ladder AB, of weight W and length $2l$, has one end A resting on rough horizontal ground. The other end B rests against a rough vertical wall. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$. The coefficient of friction between the ladder and the ground is μ. Friction is limiting at both A and B. The ladder is at an angle θ to the ground, where $\tan\theta = \frac{5}{3}$. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

Find the value of μ.



(9)



$\tan\theta = \frac{5}{3}$

$F_B = \frac{1}{3} R_B$

$F_A = \mu R_A$

$\updownarrow$ $\quad R_A + F_B = W$

$R_A + \frac{1}{3} R_B = W \quad (1)$

$\leftrightarrow$ $\quad R_B = F_A$

$R_B = \mu R_A \quad (2)$

Moments about B

$$F_A \times 2L\sin\theta + WL\cos\theta = R_A 2L\cos\theta$$

$$2F_A\sin\theta + W\cos\theta = 2R_A\cos\theta$$

$$2\mu R_A\sin\theta + W\cos\theta = 2R_A\cos\theta \quad (3)$$

From (1) and (2) $\quad R_A + \frac{\mu R_A}{3} = W$

$$R_A\left(1 + \frac{\mu}{3}\right) = W$$

Sub for w in 3

$$2\mu R_A \sin\theta + R_A\left(1+\frac{\mu}{3}\right)\cos\theta = 2R_A\cos\theta$$

$$2\mu\sin\theta + \left(1+\frac{\mu}{3}\right)\cos\theta = 2\cos\theta$$

$$2\mu\frac{5}{\sqrt{34}} + \left(1+\frac{\mu}{3}\right)\frac{3}{\sqrt{34}} = 2\times\frac{3}{\sqrt{34}}$$

$$10\mu + 3 + \mu = 6$$

$$11\mu = 3$$

$$\mu = \frac{3}{11}$$