Exponential Equations

C2 Exponentials & Logs: Exponential Equations *PhysicsAndMathsTutor.com*

1. (a) Find, to 3 significant figures, the value of x for which $5^x = 7$.

(2)

(b) Solve the equation $5^{2x} - 12(5^x) + 35 = 0$.

(4)
(Total 6 marks)

a)

$$5^x = 7$$
$$\ln(5^x) = \ln 7$$
$$x \ln 5 = \ln 7$$
$$x = \frac{\ln 7}{\ln 5}$$
$$x = 1.21$$

b)

$$5^{2x} - 12(5^x) + 35 = 0$$
$$(5^x - 5)(5^x - 7) = 0$$
$$\Rightarrow 5^x = 5 \quad \text{or} \quad 5^x = 7$$
$$\Rightarrow x = 1 \qquad \Rightarrow x = 1.21$$

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2. Solve the equation

$$5^x = 17,$$

giving your answer to 3 significant figures.

(Total 3 marks)

$$\log_{10} 5^x = \log_{10} 17$$

$x \log_{10} 5 = \log_{10} 17$

$x = \dfrac{\log_{10} 17}{\log_{10} 5} = 1.76$

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3. Solve

(a) $5^x = 8$, giving your answers to 3 significant figures,

(3)

(b) $\log_2(x + 1) - \log_2 x = \log_2 7$.

(3)
(Total 6 marks)

b) $\log_2(x+1) - \log_2 x = \log_2 7$

$\log_2\left(\dfrac{x+1}{x}\right) = \log_2 7$

$\Rightarrow \dfrac{x+1}{x} = 7$

$x + 1 = 7x$

$1 = 6x$

$x = \dfrac{1}{6}$

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4. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $3^x = 5$,

(3)

(b) $\log_2 (2x + 1) - \log_2 x = 2$.

(4)
(Total 7 marks)

a) $\ln 3^x = \ln 5$

$x \ln 3 = \ln 5$

$x = \dfrac{\ln 5}{\ln 3} = 1.46$

b) $\log_2(2x+1) - \log_2 x = 2$

$$\log_2\left(\frac{2x+1}{x}\right) = 2$$

$$\frac{2x+1}{x} = 2^2$$

$$\frac{2x+1}{x} = 4$$

$$2x+1 = 4x$$

$$1 = 2x$$

$$x = \frac{1}{2}$$

3. Solve

(a) $5^x = 8$, giving your answers to 3 significant figures,

(3)

(b) $\log_2(x+1) - \log_2 x = \log_2 7$.

(3)
(Total 6 marks)

b) $\log_2\left(\frac{x+1}{x}\right) = \log_2 7$

$$\frac{x+1}{x} = 7$$

$$x+1 = 7x$$

$$1 = 6x$$

$$x = \frac{1}{6}$$