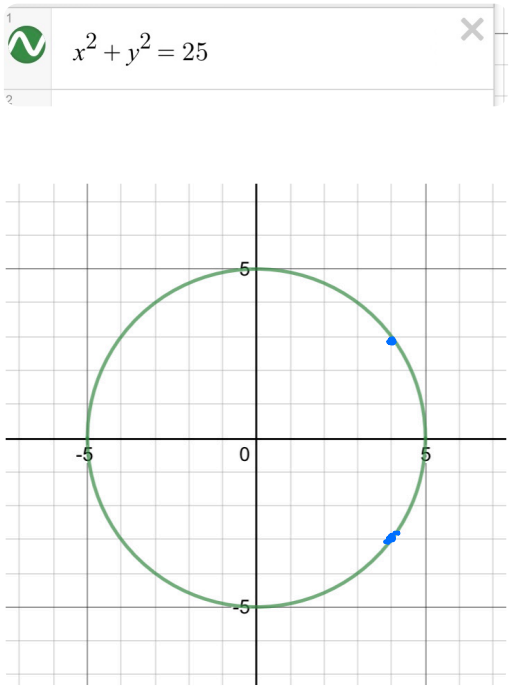


Find eqn of tangent at (4,3)

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$

$$y = \pm(25 - x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \pm\frac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x)$$

$$= \pm\frac{x}{\sqrt{25 - x^2}}$$

$$= \pm\frac{4}{3} \quad \text{-ve reqd}$$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{4}{3}(x - 4)$

$y - 3 = -\frac{4}{3}x + \frac{16}{3}$ $\quad y = -\frac{4}{3}x + \frac{25}{3}$

Implicit Differentiation

$$x^2 + y^2 = 25$$

d w.r.t. x $\quad 2x + 2y\frac{dy}{dx} = 0$

$$\frac{d}{dx} = \frac{d}{dy}\frac{dy}{dx}$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

At (4,3) $\quad \frac{dy}{dx} = -\frac{4}{3}$

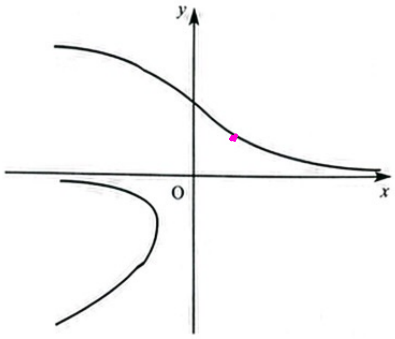


Figure 4.7

$y^3 + xy = 2$

Find eqn of tgt at $(1,1)$

d w.r.t. x

$$3y^2\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(3y^2 + x) = -y$$

$$\frac{dy}{dx} = \frac{-y}{3y^2 + x}$$

At $(1,1)$ $\frac{dy}{dx} = \frac{-1}{4}$

$y - y_1 = m(x - x_1)$

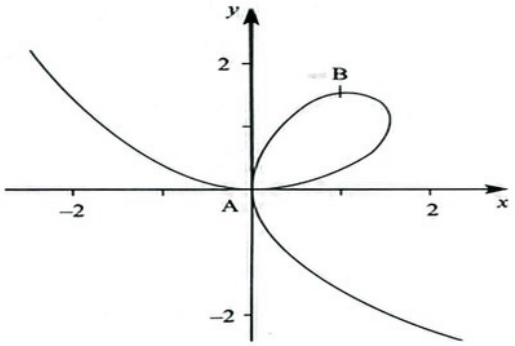
$y - 1 = -\frac{1}{4}(x - 1)$

$y - 1 = -\frac{1}{4}x + \frac{1}{4}$

$y = -\frac{1}{4}x + \frac{5}{4}$

$4y = -x + 5$

$x + 4y - 5 = 0$



$x^3 + y^3 = 3xy$

Find stationary points

d w r t x

$3x^2 + 3y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$

$x^2 + y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$

At st pt $\frac{dy}{dx} = 0$ $\Rightarrow$ $x^2 + 0 = 0 + y$

$x^2 = y$

Solve $\begin{cases} x^3 + y^3 = 3xy & (1) \\ x^2 = y & (2) \end{cases}$

Sub for y in (1)

$x^3 + x^6 = 3x^3$

$x^6 - 2x^3 = 0$

$x^3(x^3 - 2) = 0$

$\Rightarrow$ $x^3 = 0$ or $x^3 - 2 = 0$

$x = 0$ $x^3 = 2$

$y = 0$ $x = \sqrt[3]{2}$ or $2^{\frac{1}{3}}$

$(0,0)$ $y = (\sqrt[3]{2})^2$ or $2^{2/3}$

---

1 Differentiate each of the following with respect to x.

(i) y^4 **(ii)** $x^2 + y^3 - 5$ **(iii)** $xy + x + y$

(iv) $\cos y$ **(v)** $e^{(y+2)}$ **(vi)** xy^3

(vii) $2x^2y^5$ **(viii)** $x + \ln y - 3$ **(ix)** $xe^y - \cos y$

(x) $x^2 \ln y$ **(xi)** $xe^{\sin y}$ **(xii)** $x \tan y - y \tan x$

2 Find the gradient of the curve $xy^3 = 5 \ln y$ at the point $(0, 1)$.

3 Find the gradient of the curve $e^{\sin x} + e^{\cos y} = e + 1$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

4 **(i)** Find the gradient of the curve $x^2 + 3xy + y^2 = x + 3y$ at the point $(2, -1)$.

(ii) Hence find the equation of the tangent to the curve at this point.

5 Find the co-ordinates of all the stationary points on the curve $x^2 + y^2 + xy = 3$.

$\frac{d}{dx} e^{\sin x}$

$\cos x \, e^{\sin x}$

Let $u = \sin x$ $\quad \frac{du}{dx} = \cos x$

$y = e^u$

$\frac{dy}{dx} = e^u \times \frac{du}{dx}$