Show $x^3 - 2x^2 - 4 = 0$

has a root between $x = 2$ and $x = 3$

$$2^3 - 2(2)^2 - 4 = 8 - 8 - 4 = -4 \quad < 0$$

$$3^3 - 2(3)^2 - 4 = 27 - 18 - 4 = +5 \quad > 0$$

Sign change between $x = 2$ and $x = 3$

function continuous so a root between 2 and 3

Rearrange $x^3 - 2x^2 - 4 = 0$

to give a formula for x

Attempt 1

$$x^3 = 2x^2 + 4$$

$$x = \sqrt[3]{2x^2 + 4}$$

Attempt 2

$$x^3 - 4 = 2x^2$$

$$\frac{x^3 - 4}{2} = x^2$$

$$\sqrt{\frac{x^2 - 4}{2}} = x$$

Let $x_{n+1} = \sqrt[3]{2x_n^2 + 4}$ Let $x_0 = 2$

$$x_1 = \sqrt[3]{2 \times 2^2 + 4} = 2.289$$

$$x_2 = \sqrt[3]{2 \times 2.289^2 + 4} = 2.437$$

$$x_3 = \sqrt[3]{2 \times 2.437^2 + 4} = 2.513$$

Let $x_{n+1} = \sqrt{\frac{x_n^2 - 4}{2}}$ Let $x_0 = 3$

$x_1 = \sqrt{\frac{3^2 - 4}{2}} = 1.581$

$x_2 = \sqrt{\frac{1.581^2 - 4}{2}}$ X

Exercise $x^3 - x - 4 = 0$

i) Show there is a root between $x = 1$ and $x = 2$

ii) Rearrange to form iterative formula

iii) Starting $x_0 = 1$, find x_1, x_2, x_3

$1^3 - 1 - 4 = -4$

$2^3 - 2 - 4 = +2$

Sign change between $x = 1$ and $x = 2$

continuous function so root between $x = 1$ and $x = 2$

$x^3 = x + 4$

$x = \sqrt[3]{x + 4}$

$x_{n+1} = \sqrt[3]{x_n + 4}$ $x_0 = 1$

$x_1 = \sqrt[3]{1 + 4} = 1.710$

$x_2 = \sqrt[3]{1.710 + 4} = 1.787$

$x_3 = \sqrt[3]{1.787 + 4} = 1.795$
