Binomial Within the Binomial

Suppose you spin a coin 10 times, what is prob you get at least 7 heads?

$$X \sim B(10, \tfrac{1}{2}) \quad (n, p)$$

$$\begin{aligned} P(X \geq 7) &= 1 - P(X \leq 6) \\ &= 1 - 0.8281 \\ &= 0.1719 \end{aligned}$$

Suppose 5 people each carried out this experiment. What is the probability at least 3 of them got at least 7 heads?

$$Y \sim B(5, 0.1719) \quad (n, p)$$

$$\begin{aligned} P(Y \geq 3) &= 1 - P(Y \leq 2) \\ &= 1 - 0.9614 \\ &= 0.0386 \end{aligned}$$

Approximating Binomial With Normal Distribution

Spin a fair coin 100 times. Find probability that you get between 45 and 55 heads inclusive

Binomial $X \sim B(100, 0.5) \quad (n, p)$

$$P(45 \leq X \leq 55)$$

$$= P(X \leq 55) - P(X \leq 44)$$

$= 0.8643 - 0.1356$

$= 0.7287$

Approximate With Normal

$X \sim B(100, 0.5)$ (n, p)

$E(X) = np = 100 \times 0.5 = 50$

$Var(X) = npq = 100 \times 0.5 \times 0.5 = 25$

Approximate with

$Y \sim N(50, 5^2)$ (μ, σ^2)

$P(45 \leq X \leq 55) \approx P(44.5 \leq Y \leq 55.5) = 0.7287$

---

7 A geologist splits rocks to look for fossils. On average 10% of the rocks selected from a particular area do in fact contain fossils.

The geologist selects a random sample of 20 rocks from this area.

(i) Find the probability that

(*A*) exactly one of the rocks contains fossils, [3]

(*B*) at least one of the rocks contains fossils. [3]

(ii) A random sample of n rocks is selected from this area. The geologist wants to have a probability of 0.8 or greater of finding fossils in at least one of the n rocks. Find the least possible value of n. [3]

(iii) The geologist explores a new area in which it is claimed that less than 10% of rocks contain fossils. In order to investigate the claim, a random sample of 30 rocks from this area is selected, and the number which contain fossils is recorded. A hypothesis test is carried out at the 5% level.

(*A*) Write down suitable hypotheses for the test. [3]

(*B*) Show that the critical region consists only of the value 0. [4]

(*C*) In fact, 2 of the 30 rocks in the sample contain fossils. Complete the test, stating your conclusions clearly. [2]

i) $X \sim B(20, 0.1)$ ($n = 20$, $p = 0.1$)

A) $P(X = 1) = 0.2702$

B) $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.9^{20}$
$= 0.8784$

---

ii) $N = 18$ $P(X \geq 1) = 1 - 0.9^{18} = 0.8499$

$N = 16$ $P(X \geq 1) = 1 - 0.9^{16} = 0.8147$

$N = 15$ $P(X \geq 1) = 1 - 0.9^{15} = 0.7941$

Least $n = 16$

---

iii) A) $H_0 : p = 0.1$
$H_1 : p < 0.1$

where p is prob random rock contains a fossil

$P(X = 0) = 0.0423 < 5\%$
$P(X \leq 1) = 0.1836 > 5\%$

B) 5% $\therefore$ 0 is only value in critical region

---

c) 2 is not in critical region so accept H_0

There is not sufficient evidence to support the view the proportion of rocks containing fossils has reduced. Accept it is still 10%
