3 Q varies inversely with $(5 - t)$. If $Q = 8$ when $t = 3$, find the following.

a Q when $t = 10$ **b** t when $Q = 16$

4 M varies inversely with t^2. If $M = 9$ when $t = 2$, find the following.

a M when $t = 3$ **b** t when $M = 1.44$

5 W is inversely proportional to $\sqrt{T}$. If $W = 6$ when $T = 16$, find the following.

a W when $T = 25$ **b** T when $W = 2.4$

6 The grant available to a section of society was inversely proportional to the number of people needing the grant. When 30 people needed a grant, they received £60 each.

a What would the grant have been if 120 people had needed one?

b If the grant had been £50 each, how many people would have received it?

7 While doing underwater tests in one part of an ocean, a team of scientists noticed that the temperature in °C was inversely proportional to the depth in kilometres. When the temperature was 6 °C, the scientists were at a depth of 4 km.

a What would the temperature have been at a depth of 8 km?

b To what depth would they have had to go to find the temperature at 2 °C?

8 A new engine was being tested, but it had serious problems. The distance it went, in km, without breaking down was inversely proportional to the square of its speed in m/s. When the speed was 12 m/s, the engine lasted 3 km.

a Find the distance covered before a breakdown, when the speed is 15 m/s.

b On one test, the engine broke down after 6.75 km. What was the speed?

9 In a balloon it was noticed that the pressure, in atmospheres, was inversely proportional to the square root of the height, in metres. When the balloon was at a height of 25 m, the pressure was 1.44 atm.

a What was the pressure at a height of 9 m?

b What would the height have been if the pressure was 0.72 atm?

10 The amount of waste which a firm produces, measured in tonnes per hour, is inversely proportional to the square root of the size of the filter beds, measured in m^2. At the moment, the firm produces 1.25 tonnes per hour of waste, with filter beds of size 0.16 m^2.

a The filter beds used to be only 0.01 m^2. How much waste did the firm produce then?

b How much waste could be produced if the filter beds were 0.75 m^2?

6 The grant available to a section of society was inversely proportional to the number of people needing the grant. When 30 people needed a grant, they received £60 each.

a What would the grant have been if 120 people had needed one?

b If the grant had been £50 each, how many people would have received it?

Let P be number of people

Let G be size of grant

$$G = \frac{k}{P}$$

$P = 30$

$G = £60$

$$60 = \frac{k}{30}$$

$$60 \times 30 = k$$

$$1800 = k$$

$$G = \frac{1800}{P}$$

a) when $P = 120$

$$G = \frac{1800}{120} = 15$$

Grant = £15

b) when $G = £50$

$$50 = \frac{1800}{P}$$

$$50P = 1800$$

$$P = \frac{1800}{50}$$

$$P = 36$$

36 people

3 Q varies inversely with $(5 - t)$. If $Q = 8$ when $t = 3$, find the following.

a Q when $t = 10$

b t when $Q = 16$

$$Q = \frac{k}{5-t}$$

$$\begin{cases} Q = 8 \\ t = 3 \end{cases}$$

$$8 = \frac{k}{5-3}$$

$$8 = \frac{k}{2}$$

$$16 = k$$

$$Q = \frac{16}{5-t}$$

a) When $t = 10$

$$Q = \frac{16}{5-10}$$

$$Q = \frac{16}{-5}$$

$$Q = -3.2$$

b) When $Q = 16$

$$16 = \frac{16}{(5-t)}$$

$$16(5-t) = 16$$

$$5 - t = \frac{16}{16}$$

$$5 - t = 1$$

$$t = 4$$

5 W is inversely proportional to $\sqrt{T}$. If $W = 6$ when $T = 16$, find the following.

a W when $T = 25$

b T when $W = 2.4$

$$W = \frac{k}{\sqrt{T}}$$

$$\begin{cases} W = 6 \\ T = 16 \end{cases}$$

$$6 = \frac{k}{\sqrt{16}}$$

$$6 = \frac{k}{4}$$

$$24 = k$$

$$W = \frac{24}{\sqrt{T}}$$

a) When $T = 25$

$$W = \frac{24}{\sqrt{25}}$$

$$W = \frac{24}{5}$$

$$W = 4.8$$

b) When $W = 2.4$

$$2.4 = \frac{24}{\sqrt{T}}$$

$$2.4\sqrt{T} = 24$$

$$\sqrt{T} = \frac{24}{2.4} = 10$$

$$T = 10^2$$

$$T = 100$$

8 A new engine was being tested, but it had serious problems. The distance it went, in km, without breaking down was inversely proportional to the square of its speed in m/s. When the speed was 12 m/s, the engine lasted 3 km.

a Find the distance covered before a breakdown, when the speed is 15 m/s.

b On one test, the engine broke down after 6.75 km. What was the speed?

Distance D Speed V

$$D = \frac{k}{V^2}$$

$$\begin{cases} D = 3 \\ V = 12 \end{cases}$$

$$3 = \frac{k}{12^2}$$

$$3 = \frac{k}{144}$$

$$D = \frac{432}{V^2}$$

$432 = k$

a) when $v = 15$

$$D = \frac{432}{15^2}$$

$$D = \frac{432}{225}$$

$$D = 1.92 \text{ km}$$

b) when $D = 6.75$

$$6.75 = \frac{432}{v^2}$$

$$6.75v^2 = 432$$

$$v^2 = \frac{432}{6.75}$$

$$v = \sqrt{\frac{432}{6.75}}$$

$$v = 8 \text{ ms}^{-1}$$

10 The amount of waste which a firm produces, measured in tonnes per hour, is inversely proportional to the square root of the size of the filter beds, measured in m^2. At the moment, the firm produces 1.25 tonnes per hour of waste, with filter beds of size 0.16 m^2.

a The filter beds used to be only 0.01 m^2. How much waste did the firm produce then?

b How much waste could be produced if the filter beds were 0.75 m^2?

Weight W Filter bed area A

$$W = \frac{k}{\sqrt{A}}$$

$W = 1.25$ tonnes

$A = 0.16 \text{ m}^2$

$$1.25 = \frac{k}{\sqrt{0.16}}$$

$$1.25 = \frac{k}{0.4}$$

$$1.25 \times 0.4 = k$$

$$0.5 = k$$

$$W = \frac{0.5}{\sqrt{A}}$$

a) When $A = 0.01\ m^2$

$$W = \frac{0.5}{\sqrt{0.01}} = \frac{0.5}{0.1} = \underline{5 \text{ tonnes}}$$

b) When $A = 0.75\ m^2$

$$W = \frac{0.5}{\sqrt{0.75}}$$

$$\underline{W = 0.577 \text{ tonnes}}$$

14 y is inversely proportional to d^2
When $d = 10$, $y = 4$

d is directly proportional to x^2
When $x = 2$, $d = 24$

Find a formula for y in terms of x.
Give your answer in its simplest form.

$y = \frac{k}{d^2}$

$4 = \frac{k}{10^2}$

$400 = k$

$d = cx^2$

$24 = c \times 2^2$

$\frac{24}{4} = c$

$6 = c$

$d = 10$

$y = 4$

$y = \frac{400}{d^2}$

$x = 2$

$d = 24$

$\underline{d = 6x^2}$

$y = \frac{400}{d^2}$

$y = \frac{400}{(6x^2)^2}$

$y = \frac{400}{36x^4}$

$y = \frac{100}{9x^4}$