

## Independent Events

Events A and B are independent when the fact that one of them occurs does not affect the probability occurs.

For independent events A and B

$$P(A \cap B) = P(A) \times P(B)$$

This is called the 'AND' rule

Let A be get a Head spinning a coin

Example

Let B be get a 4 rolling a dice

$$P(A) = \frac{1}{2} \quad P(B) = \frac{1}{6}$$

$$P(A \cap B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

See where this comes from

H 1	T 1
H 2	T 2
H 3	T 3
<span style="border: 1px solid red;">H 4</span>	T 4
H 5	T 5
H 6	T 6

12 equally likely outcomes

$$\text{so } P(H_4) = \frac{1}{12}$$

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## Pack of Cards (Facts)

52 cards      4 suits      Heart, Diamond, Spade, Club  
2 colours      Red      Red      Black      Black

Each suit 13 cards      A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

Examples Pick card at random

$$P(\text{Heart}) = \frac{13}{52} = \frac{1}{4}$$

$$P(\text{Red}) = \frac{26}{52} = \frac{1}{2}$$

$$P(\text{Queen}) = \frac{4}{52} = \frac{1}{13}$$

Let A be event pick Queen

Let B be event pick Spade

$$P(A) = \frac{1}{13}$$

$$P(B) = \frac{1}{4}$$

$$P(\text{Queen and Spade}) = P(A \cap B) = \frac{1}{13} \times \frac{1}{4} = \frac{1}{52}$$

$$P(A) \times P(B) = \frac{1}{13} \times \frac{1}{4} = \frac{1}{52}$$

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Let C be event pick Red card

Let D be event pick Heart

$$P(C) = \frac{1}{2}$$

$$P(D) = \frac{1}{4}$$

These affect each other - they are not independent  $P(C \cap D) = \frac{1}{4}$

$$\text{but } P(C) \times P(D) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

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This is the test for independence

A, B are independent if  $P(A) \times P(B) = P(A \cap B)$

Spin a coin 3 times - What is Prob (HHH)

HHH

HHT

HTH

HTT

T HH

T HT

T TH

TTT

$$P(HHH) = \frac{1}{8}$$

Each spin is independent

so  $P(H \cap H \cap H)$

$$= P(H) \times P(H) \times P(H)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(2H, 1T) = \frac{3}{8}$$

$$P(1H, 2T) = \frac{3}{8}$$

$$P(3T) = \frac{1}{8}$$

Notice outcomes  
are not equally  
likely

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