Normal Distribution - Hypothesis Testing - MEI Questions

Jun 05

4 **(a)** The selling prices of semi-detached houses in the suburbs of a particular city are known to be Normally distributed with mean £166 500 and standard deviation £14 200. A householder on one large estate claims that houses on her estate have a higher mean selling price. The selling prices of six randomly selected houses on her estate are

£180 000, £152 000, £156 500, £172 000, £189 000, £169 000.

(i) State suitable null and alternative hypotheses to test her claim. [2]

(ii) Carry out the test at the 5% level of significance, stating your conclusions clearly. You may assume that the standard deviation of the selling prices of houses on this estate is £14 200. [6]

Jan 06

The mean travel time for pupils who travel by bus is known to be 18.3 minutes. A survey is carried out to determine whether the mean travel time to school by car is different from 18.3 minutes. In the survey, 20 pupils who travel by car are selected at random. Their mean travel time is found to be 22.4 minutes.

(iv) Assuming that car travel times are Normally distributed with standard deviation 8.0 minutes, carry out a test at the 10% level, stating your hypotheses and conclusion clearly. [7]

(v) Comment on the suggestion that pupils should use a bus if they want to get to school quickly. [2]

Jan 08

(b) Over a long period it has been determined that the mean score of students in a particular English module is 67.4 and the standard deviation is 8.9. A new teaching method is introduced with the aim of improving the results. A random sample of 12 students taught by the new method is selected. Their mean score is found to be 68.3. Carry out a test at the 10% level to investigate whether the new method appears to have been successful. State carefully your null and alternative hypotheses. You should assume that the scores are Normally distributed and that the standard deviation is unchanged. [7]

Jan 09

(ii) It is known that the diameter of marigold flowers is Normally distributed with mean 47 mm and standard deviation 8.5 mm. A certain fertiliser is expected to cause flowers to have a larger mean diameter, but without affecting the standard deviation. A large number of marigolds are grown using this fertiliser. The diameters of a random sample of 50 of the flowers are measured and the mean diameter is found to be 49.2 mm. Carry out a hypothesis test at the 1% significance level to check whether flowers grown with this fertiliser appear to be larger on average. Use hypotheses $H_0 : \mu = 47$, $H_1 : \mu > 47$, where μ mm represents the mean diameter of all marigold flowers grown with this fertiliser. **[5]**

Jun 06

2 The head circumference of 3-year-old boys is known to be Normally distributed with mean 49.7 cm and standard deviation 1.6 cm.

(i) Find the probability that the head circumference of a randomly selected 3-year-old boy will be

(A) over 51.5 cm,

(B) between 48.0 and 51.5 cm. [5]

(ii) Four 3-year-old boys are selected at random. Find the probability that exactly one of them has head circumference between 48.0 and 51.5 cm. [3]

(iii) The head circumference of 3-year-old girls is known to be Normally distributed with mean μ and standard deviation σ. Given that 60% of 3-year-old girls have head circumference below 49.0 cm and 30% have head circumference below 47.5 cm, find the values of μ and σ. [4]

A nutritionist claims that boys who have been fed on a special organic diet will have a larger mean head circumference than other boys. A random sample of ten 3-year-old boys who have been fed on this organic diet is selected. It is found that their mean head circumference is 50.45 cm.

(iv) Using the null and alternative hypotheses H_0: $\mu = 49.7$ cm, H_1: $\mu > 49.7$ cm, carry out a test at the 10% significance level to examine the nutritionist's claim. Explain the meaning of μ in these hypotheses. You may assume that the standard deviation of the head circumference of organically fed 3-year-old boys is 1.6 cm. [6]

Jan 07

2 **(a)** A farmer grows Brussels sprouts. The diameter of sprouts in a particular batch, measured in mm, is Normally distributed with mean 28 and variance 16. Sprouts that are between 24 mm and 33 mm in diameter are sold to a supermarket.

(i) Find the probability that the diameter of a randomly selected sprout will be within this range. [4]

(ii) The farmer sells the sprouts in this range to the supermarket for 10 pence per kilogram. The farmer sells sprouts under 24 mm in diameter to a frozen food factory for 5 pence per kilogram. Sprouts over 33 mm in diameter are thrown away. Estimate the total income received by the farmer for the batch, which weighs 25 000 kg. [3]

(iii) By harvesting sprouts earlier, the mean diameter for another batch can be reduced to k mm. Find the value of k for which only 5% of the sprouts will be above 33 mm in diameter. You may assume that the variance is still 16. [3]

(b) The farmer also grows onions. The weight in kilograms of the onions is Normally distributed with mean 0.155 and variance 0.005. He is trying out a new variety, which he hopes will yield a higher mean weight. In order to test this, he takes a random sample of 25 onions of the new variety and finds that their total weight is 4.77 kg. You should assume that the weight in kilograms of the new variety is Normally distributed with variance 0.005.

(i) Write down suitable null and alternative hypotheses for the test in terms of μ. State the meaning of μ in this case. [2]

(ii) Carry out the test at the 1% level. [6]

Jun 07

1 The random variable X represents the time taken in minutes for a haircut at a barber's shop. X is Normally distributed with mean 11 and standard deviation 3.

(i) Find $P(X < 10)$. [4]

(ii) Find the probability that exactly 3 out of 8 randomly selected haircuts take less than 10 minutes. [3]

(iii) Use a suitable approximating distribution to find the probability that at least 50 out of 100 randomly selected haircuts take less than 10 minutes. [4]

A new hairdresser joins the shop. The shop manager suspects that she takes longer on average than the other staff to do a haircut. In order to test this, the manager records the time taken for 25 randomly selected cuts by the new hairdresser. The mean time for these cuts is 12.34 minutes. You should assume that the time taken by the new hairdresser is Normally distributed with standard deviation 3 minutes.

(iv) Write down suitable null and alternative hypotheses for the test. [3]

(v) Carry out the test at the 5% level. [5]

Jun 08

3 A company has a fleet of identical vans. Company policy is to replace all of the tyres on a van as soon as any one of them is worn out. The random variable X represents the number of miles driven before the tyres on a van are replaced. X is Normally distributed with mean 27 500 and standard deviation 4000.

(i) Find $P(X > 25\,000)$. [4]

(ii) 10 vans in the fleet are selected at random. Find the probability that the tyres on exactly 7 of them last for more than 25 000 miles. [3]

(iii) The tyres of 99% of vans last for more than k miles. Find the value of k. [3]

A tyre supplier claims that a different type of tyre will have a greater mean lifetime. A random sample of 15 vans is fitted with these tyres. For each van, the number of miles driven before the tyres are replaced is recorded. A hypothesis test is carried out to investigate the claim. You may assume that these lifetimes are also Normally distributed with standard deviation 4000.

(iv) Write down suitable null and alternative hypotheses for the test. [3]

(v) For the 15 vans, it is found that the mean lifetime of the tyres is 28 630 miles. Carry out the test at the 5% level. [5]

Jun 09

3 Intensity of light is measured in lumens. The random variable X represents the intensity of the light from a standard 100 watt light bulb. X is Normally distributed with mean 1720 and standard deviation 90. You may assume that the intensities for different bulbs are independent.

(i) Show that $P(X < 1700) = 0.4121$. **[4]**

(ii) These bulbs are sold in packs of 4. Find the probability that the intensities of exactly 2 of the 4 bulbs in a randomly chosen pack are below 1700 lumens. **[3]**

(iii) Use a suitable approximating distribution to find the probability that the intensities of at least 20 out of 40 randomly selected bulbs are below 1700 lumens. **[5]**

A manufacturer claims that the average intensity of its 25 watt low energy light bulbs is 1720 lumens. A consumer organisation suspects that the true figure may be lower than this. The intensities of a random sample of 20 of these bulbs are measured. A hypothesis test is then carried out to check the claim.

(iv) Write down a suitable null hypothesis and explain briefly why the alternative hypothesis should be $H_1 : \mu < 1720$. State the meaning of μ. **[3]**

(v) Given that the standard deviation of the intensity of such bulbs is 90 lumens and that the mean intensity of the sample of 20 bulbs is 1703 lumens, carry out the test at the 5% significance level. **[5]**

Jan 10

The council is also investigating the weight of rubbish in domestic dustbins. In 2008 the average weight of rubbish in bins was 32.8 kg. The council has now started a recycling initiative and wishes to determine whether there has been a reduction in the weight of rubbish in bins. A random sample of 50 domestic dustbins is selected and it is found that the mean weight of rubbish per bin is now 30.9 kg, and the standard deviation is 3.4 kg.

(iv) Carry out a test at the 5% level to investigate whether the mean weight of rubbish has been reduced in comparison with 2008. State carefully your null and alternative hypotheses. **[8]**