Standard Differentials

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

Chain Rule

Ex1 Let $y = (2x^2+3x-5)^3$ Find $\frac{dy}{dx}$

Let $u = 2x^2+3x-5$

$\Rightarrow \frac{du}{dx} = 4x+3$

$y = u^3$

$\Rightarrow \frac{dy}{du} = 3u^2$

Chain Rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\frac{dy}{dx} = 3u^2 \times (4x+3) = 3(2x^2+3x-5)^2(4x+3)$$

This a formal approach to the Chain Rule

Informal approach

$\frac{d}{dx}$ of a bracket to the power n

$= n(\quad)^{n-1} \times$ differential of the bracket

Ex 2 $y = \sin 4x$ Find $\frac{dy}{dx}$

Let $u = 4x$ $y = \sin u$

$\frac{du}{dx} = 4$ $\frac{dy}{du} = \cos u$

Chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \cos u \times 4$

$= 4 \cos 4x$

Ex 3 $y = \cos(x^4)$ Find $\frac{dy}{dx}$

Let $u = x^4$ $y = \cos u$

$\frac{du}{dx} = 4x^3$ $\frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\sin u \times 4x^3 = -4x^3 \sin(x^4)$

Ex 3

Let $y = e^{x^2+x}$ Find $\frac{dy}{dx}$

Let $u = x^2+x$ $\quad y = e^u$

$\frac{du}{dx} = 2x+1$ $\quad \frac{dy}{du} = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \times (2x+1)$$

$$= (2x+1)e^{x^2+x}$$

Informally $\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$

$$\frac{d}{dx} \sin(f(x)) = f'(x) \cos(f(x))$$

$$\frac{d}{dx} \cos(f(x)) = -f'(x) \sin(f(x))$$

Ex 4

Let $y = \ln(x^2+3x+1)$ Find $\frac{dy}{dx}$

Let $u = x^2+3x+1$ $\quad y = \ln u$

$\frac{du}{dx} = 2x+3$ $\quad \frac{dy}{du} = \frac{1}{u}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \times (2x+3) = \frac{2x+3}{x^2+3x+1}$$

In general $\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$

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1b $\frac{d}{dx}(3-2x^2)^{-5} = -5(3-2x^2)^{-6}(-4x)$

$= \frac{20x}{(3-2x^2)^6}$

1d) $\frac{d}{dx}(6x+x^2)^7 = 7(6x+x^2)^6(2x)$

$= 14x(6x+x^2)^6$

1f) $\frac{d}{dx}\sqrt{7-x} = \frac{d}{dx}(7-x)^{\frac{1}{2}}$

$= \frac{1}{2}(7-x)^{-\frac{1}{2}}(-1)$

$= -\frac{1}{2}(7-x)^{-\frac{1}{2}}$

or $-\frac{1}{2\sqrt{7-x}}$

1h) $\frac{d}{dx}3(8-x)^{-6} = 3x-6(8-x)^{-7}(-1)$

$= 18(8-x)^{-7}$

2b) $\frac{d}{dx}\cos(2x-1) = -2\sin(2x-1)$

2d) $\frac{d}{dx}(\sin x+\cos x)^5 = 5(\sin x+\cos x)^4(\cos x-\sin x)$

2f) $\frac{d}{dx}\ln\sin x = \frac{\cos x}{\sin x}$

2h) $\frac{d}{dx}\cos(e^{2x}+3) \quad -2e^{2x}\sin(e^{2x}+3)$

4) $y = (5-2x)^3$

$\frac{dy}{dx} = 3(5-2x)^2(-2)$

$\frac{dy}{dx} = -6(5-2x)^2$

when $x=1$ $\frac{dy}{dx} = -6(5-2)^2 = -54$

when $x=1$, $y=(5-2)^3 = 27$

$m = -54 \qquad (x_1, y_1) = (1, 27)$

$y-y_1 = m(x-x_1)$

$y-27 = -54(x-1)$

$y = -54x+54+27$

$y = -54x+81$