Roots of Polynomials and Standard Series

Q1

4 Use standard series formulae to find $\sum_{r=1}^{n} r(r^2+1)$, factorising your answer as far as possible. [6]

Q2

5 The roots of the cubic equation $2x^3 - 3x^2 + x - 4 = 0$ are α, β and γ.

Find the cubic equation whose roots are $2\alpha+1, 2\beta+1$ and $2\gamma+1$, expressing your answer in a form with integer coefficients. [7]

Q3

5 The roots of the cubic equation $x^3 + 3x^2 - 7x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 3α, 3β and 3γ, expressing your answer in a form with integer coefficients. [6]

Q4

4 Using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{n} [(r+1)(r-2)] = \frac{1}{3}n(n^2-7)$. [6]

Q5

5 The equation $x^3 + px^2 + qx + r = 0$ has roots α, β and γ, where

$$\alpha+\beta+\gamma = 3,$$
$$\alpha\beta\gamma = -7,$$
$$\alpha^2+\beta^2+\gamma^2 = 13.$$

(i) Write down the values of p and r. [2]

(ii) Find the value of q. [3]

Q6

6 The roots of the cubic equation $2x^3 + x^2 - 3x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 2α, 2β and 2γ, expressing your answer in a form with integer coefficients. [5]

Q7

10 **(i)** Using the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, prove that

$$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]