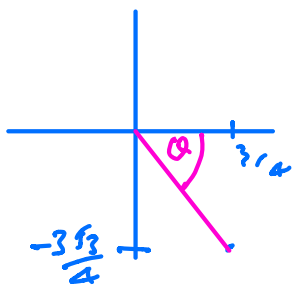
Exercise 2C

2 a) $\dfrac{3}{1+\sqrt{3}i} \times \dfrac{1-\sqrt{3}i}{1-\sqrt{3}i}$

$= \dfrac{3-3\sqrt{3}i}{1^2+\sqrt{3}^2} = \dfrac{3}{4} - \dfrac{3\sqrt{3}}{4}i$

$|r| = \sqrt{\left(\frac{3}{4}\right)^2 + \left(-\frac{3\sqrt{3}}{4}\right)^2} = \sqrt{\frac{9}{16} + \frac{27}{16}}$

$= \sqrt{\frac{36}{16}} = \frac{3}{2}$



$\theta = \tan^{-1}\left(\dfrac{\frac{3\sqrt{3}}{4}}{\frac{3}{4}}\right)$

$\theta = \tan^{-1}\left(\dfrac{\cancel{3}\sqrt{3}}{\cancel{4}} \times \dfrac{\cancel{4}}{\cancel{3}}\right)$

$\theta = \tan^{-1}\sqrt{3}$

$\theta = \frac{\pi}{3}$

argument $= -\frac{\pi}{3}$

In modarg form

$\dfrac{3}{1+\sqrt{3}i} = \frac{3}{2}\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

3 a) $5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= 5\cos\frac{\pi}{2} + i \times 5\sin\frac{\pi}{2} = 5i$

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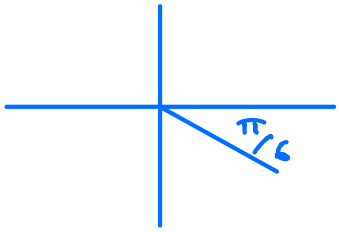
5) $|z| = 7 \quad \arg z = \frac{11\pi}{6}$

$7\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = 7\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$



$7\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= \frac{7\sqrt{3}}{2} - \frac{7i}{2}$

Exercise

2 b, c 3 b, c 4 6

Multiplying and Dividing in Mod Arg Form

For any two complex numbers z_1, z_2

$$|z_1 z_2| = |z_1| \times |z_2|$$

$$\arg(z_1 z_2) = \arg z_1 + \arg z_2$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$$

Exercise 2D

a) $z_1 = 5\left(\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right)$

$z_2 = 6\left(\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right)$

i) $|z_1 z_2| = |z_1||z_2| = 5 \times 6 = 30$

$\arg(z_1 z_2) = \arg z_1 + \arg z_2$

$= \frac{3\pi}{8} + \frac{7\pi}{8} = \frac{10\pi}{8} = -\frac{6\pi}{8}$

$z_1 z_2 = 30\left(\cos\left(-\frac{6\pi}{8}\right) + i\sin\left(-\frac{6\pi}{8}\right)\right)$

$= 30\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

$$= -\frac{30}{\sqrt{2}} - \frac{30}{\sqrt{2}} i$$

$$= -15\sqrt{2} - 15\sqrt{2} i$$

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3 a) $(\cos 2\theta + i \sin 2\theta)(\cos 3\theta + i \sin 3\theta)$

$$= \cos 5\theta + i \sin 5\theta$$

3c) $3\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) \times 2\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$

$$= 6\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

$$= 6\left(\frac{1}{2} + i \frac{\sqrt{3}}{2}\right)$$

$$= 3 + 3\sqrt{3} i$$

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4 a) $\dfrac{\cos 5\theta + i \sin 5\theta}{\cos 2\theta + i \sin 2\theta} = \cos 3\theta + i \sin 3\theta$

4d) $\dfrac{\cos 2\theta - i \sin 2\theta}{\cos 3\theta + i \sin 3\theta} = \dfrac{\cos(-2\theta) + i \sin(-2\theta)}{\cos 3\theta + i \sin 3\theta}$

$$= \cos(-5\theta) + i \sin(-5\theta)$$

HWK

Try one part of each question in Ex 2D