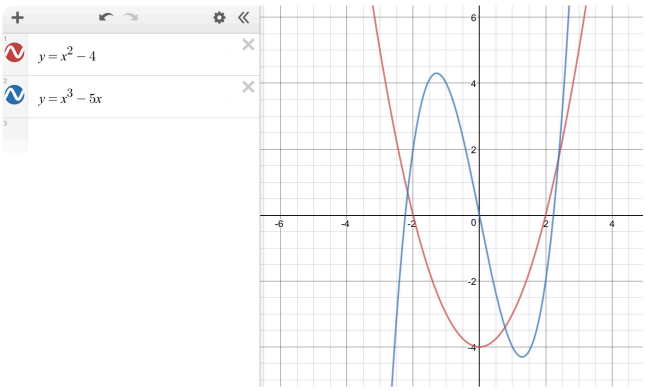
Area Between Two Curves



Find area between
$y = x^2 - 4$ and
$y = x^3 - 5x$
and the lines
$x = -2$ and $x = 0$

In this region $f(x) = x^3 - 5x$
is above $g(x) = x^2 - 4$

Area is given by $\text{Area} = \int_{-2}^{0} (f(x) - g(x))\,dx$

$$= \int_{-2}^{0} (x^3 - 5x - x^2 + 4)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} + 4x \right]_{-2}^{0}$$

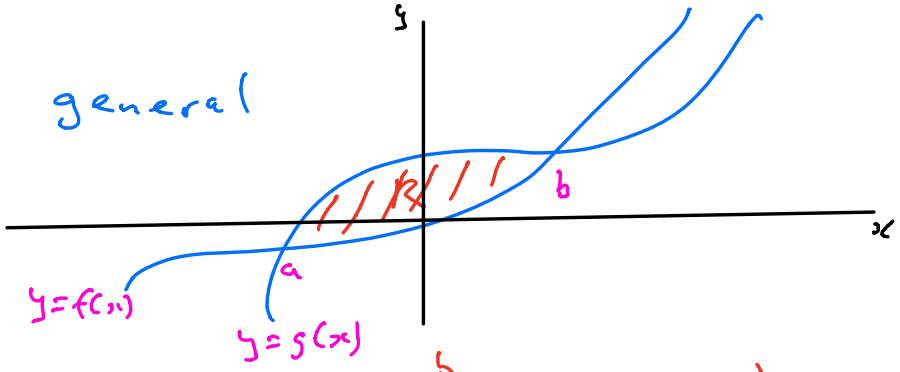
$$= \left(0 - 0 - 0 + 0 \right) - \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - \frac{5(-2)^2}{2} + 4(-2) \right)$$

$$= 0 - \left(+4 + \frac{8}{3} - 10 - 8 \right)$$

$$= 0 - 4 - \frac{8}{3} + 10 + 8$$

$$= 11\frac{1}{3} \text{ units}^2$$

In general



$$\text{Area } R = \int_a^b (g(x) - f(x))\,dx$$

Top curve − bottom curve and fact that part of region is below x-axis does not affect the answer

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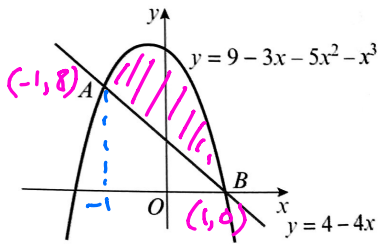
Exercise 13G

P **3** The diagram shows a sketch of part of the curve with equation $y = 9 - 3x - 5x^2 - x^3$ and the line with equation $y = 4 - 4x$. The line cuts the curve at the points $A(-1, 8)$ and $B(1, 0)$.

Find the area of the shaded region between AB and the curve.



Option 1

$$\text{Area under curve} = \int_{-1}^{1} (9 - 3x - 5x^2 - x^3)\,dx$$

$$= \left[9x - \frac{3x^2}{2} - \frac{5x^3}{3} - \frac{x^4}{4} \right]_{-1}^{1}$$

$$= \left(9 - \frac{3}{2} - \frac{5}{3} - \frac{1}{4}\right) - \left(-9 - \frac{3}{2} + \frac{5}{3} - \frac{1}{4}\right)$$

$$= 9 - \frac{3}{2} - \frac{5}{3} - \frac{1}{4} + 9 + \frac{3}{2} - \frac{5}{3} + \frac{1}{4}$$

$= 18 - \frac{10}{3} = \frac{44}{3}$

Area under line $= \frac{1}{2}$ base $\times$ Height

$= \frac{1}{2} \times 2 \times 8$

$= 8$

Area between curve and line

$= \frac{44}{3} - 8$

$= \frac{20}{3}$ units^2

Option 2 $\quad$ Area $= \int_{-1}^{1} \left(9 - 3x - 5x^2 - x^3 - (4 - 4x)\right) dx$

$= \int_{-1}^{1} \left(5 + x - 5x^2 - x^3\right) dx$

$= \left[5x + \frac{x^2}{2} - \frac{5x^3}{3} - \frac{x^4}{4}\right]_{-1}^{1}$

$= \left(5 + \frac{1}{2} - \frac{5}{3} - \frac{1}{4}\right) - \left(-5 + \frac{1}{2} + \frac{5}{3} - \frac{1}{4}\right)$

$= 5 + \frac{1}{2} - \frac{5}{3} - \frac{1}{4} + 5 - \frac{1}{2} - \frac{5}{3} + \frac{1}{4}$

$= 10 - \frac{10}{3} = \frac{20}{3}$ unit^2
