Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Sample Assessment Materials

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

Solutions

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A van of mass 750 kg is moving along a straight horizontal road.

At the instant when the speed of the van is $v\,\text{m s}^{-1}$, the resistance to the motion of the van is modelled as a force of magnitude $(200 + v^2)$ N.

When the engine of the van is working at a constant rate of 12 kW, and the van is moving at a constant speed,

(a) show that the van must be moving at $20\,\text{m s}^{-1}$, justifying your answer.

(4)

Later on, the van is moving up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$

At the instant when the speed of the van is $v\,\text{m s}^{-1}$, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $(200 + v^2)$ N.

The engine of the van is now working at a constant rate of 15 kW.

(b) Find the acceleration of the van at the instant when $v = 10$

(5)

a) Power $= Fv = 12000$ W

$F = \frac{12000}{v}$

At constant speed F = resistive force

$\Rightarrow \frac{12000}{v} = 200 + v^2$

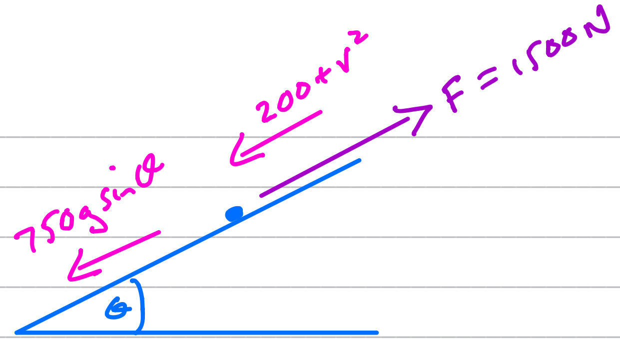
$12000 = 200v + v^3$

$v^3 + 200v - 12000 = 0$

By calc $v = 20\ \text{ms}^{-1}$ (other roots not real)

Verify $20^3 + 200(20) - 12000 = 0$ ✓

Question 1 continued



Power $= Fv = 15000$ W

When $v = 10\,\text{ms}^{-1}$ $\quad F = \dfrac{15000}{10} = 1500$ N

N2L

$$1500 - (200 + 10^2) - 750 \times 9.8 \times \frac{1}{15} = 750a$$

$$710 = 750a$$

$$a = \frac{710}{750} = 0.947\ \text{ms}^{-2}$$

to 3 s.f.

(Total for Question 1 is 9 marks)

2.

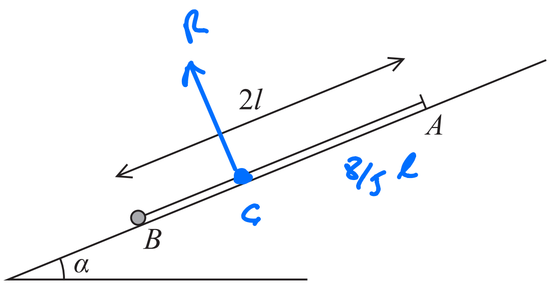


Figure 1

One end of a light elastic string, of natural length l and modulus of elasticity $\frac{3}{4}mg$, is attached to a particle of mass m. The other end of the string is attached to a fixed point A on a rough inclined plane. The plane is inclined at angle α to the horizontal, where

$\tan\alpha = \frac{5}{12}$

Initially the particle is held at the point B on the plane, where $AB = 2l$ and B lies below A on the line of greatest slope through A, as shown in Figure 1.

The particle is released from rest at B and first comes to instantaneous rest at the point C, where C is between A and B and $AC = \frac{8}{5}l$.

Find the coefficient of friction between the particle and the plane.

(6)

Work done by string = work against gravity + work against friction

$$\left(\frac{l^2}{2l} - \frac{\left(\frac{3}{5}l\right)^2}{2l}\right) \times \frac{3mg}{4} = mg\,\frac{2}{5}l\sin\alpha + \mu R\,\frac{2}{5}l$$

$$\frac{\frac{16}{25}l^2}{2l} \times \frac{3mg}{4} = mg\,\frac{2}{5}l \times \frac{5}{13} + \mu\, mg\,\frac{12}{13} \times \frac{2}{5}l$$

$$\frac{6}{25} = \frac{2}{13} + \frac{24}{65}\mu$$

$$\mu = \frac{\frac{6}{25} - \frac{2}{13}}{\frac{24}{65}} = \frac{7}{30} \approx 0.233$$

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3. A particle, P, of mass 3 kg is moving with velocity $(2\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{I}$ of magnitude $\sqrt{130}$ N s. Immediately after receiving the impulse, P is moving with velocity $(-\mathbf{i} + \lambda\mathbf{j})\,\text{m s}^{-1}$, where λ is a positive constant.

(a) Find $\mathbf{I}$, giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$.

(6)

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

(b) Find the value of θ

(3)

a) Impulse = Change in Momentum

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = 3\begin{pmatrix} -1 \\ \lambda \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\Rightarrow \quad I_1 = -3 - 6 = -9$$
$$I_2 = 3\lambda - 3$$

$$|\underline{I}| = \sqrt{130} \text{ Ns}$$

$$\Rightarrow \sqrt{(-9)^2 + (3\lambda - 3)^2} = \sqrt{130}$$

$$\Rightarrow 81 + 9\lambda^2 - 18\lambda + 9 = 130$$

$$9\lambda^2 - 18\lambda - 40 = 0$$

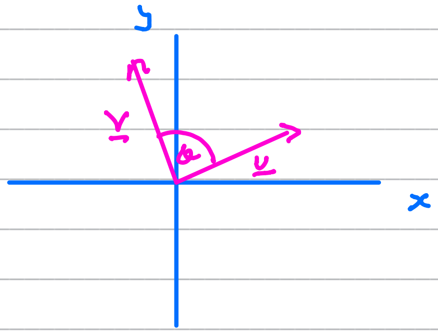
By calc $\lambda = \frac{10}{3}$ or $\cancel{\lambda = -\frac{4}{3}}$

since $\lambda > 0$

$$\therefore I_2 = 3\left(\frac{10}{3}\right) - 3 = 7$$

$$\underline{\underline{I}} = \begin{pmatrix} -9 \\ 7 \end{pmatrix} = -9\underline{i} + 7\underline{j} \text{ Ns}$$

Question 3 continued



$$\underline{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} -1 \\ \frac{10}{3} \end{pmatrix}$$

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ \frac{10}{3} \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right| \left|\begin{pmatrix} -1 \\ \frac{10}{3} \end{pmatrix}\right|}$$

$$\cos\theta = \frac{-2 + \frac{10}{3}}{\sqrt{5}\sqrt{\frac{109}{9}}} = 0.17134$$

$$\theta = \cos^{-1}(0.17134)$$

$$\theta = 80.1^\circ$$

(Total for Question 3 is 9 marks)

4.

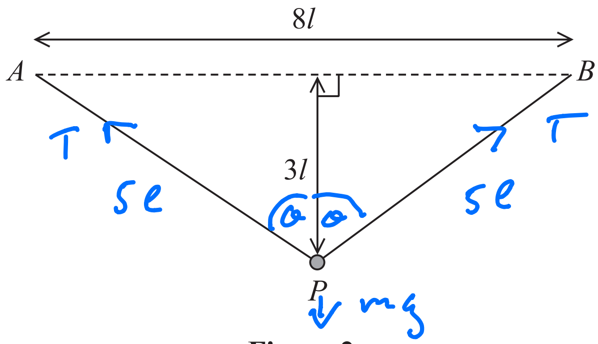


Figure 2

A light elastic string, of natural length $8l$ and modulus of elasticity kmg, has its ends attached to two points A and B, where $AB = 8l$ and AB is horizontal.

A pebble, P, of mass m is attached to the midpoint of the string. The pebble rests in equilibrium at a distance $3l$ vertically below AB, as shown in Figure 2. The pebble is modelled as a particle, and air resistance is modelled as negligible.

(a) Show that $k = \dfrac{10}{3}$

(4)

The pebble is pulled vertically downwards from its equilibrium position until the total length of the string is $\dfrac{40}{3}l$. The pebble is released from rest.

(b) Find the acceleration of P at the instant it is released from rest.

(3)

At the instant the pebble crosses the line AB, the pebble has speed v.

(c) Find v.

(3)

In an experiment, when the natural length of the string was 2 m, it was found that the speed of P at the instant when it crossed the line AB was $1.5\,\text{m s}^{-1}$.

(d) Considering the model, suggest a reason, other than air resistance, why the model and the experiment give different values.

(1)

a) $\updownarrow$ $2T\cos\theta = mg$

3,4,5 Δs

so $\cos\theta = \dfrac{3}{5}$

$2T \times \dfrac{3}{5} = mg$

$\Rightarrow \quad T = \dfrac{5mg}{6}$ (1)

Question 4 continued

But $T = \dfrac{\lambda x}{8l} = \dfrac{kmgx}{l}$

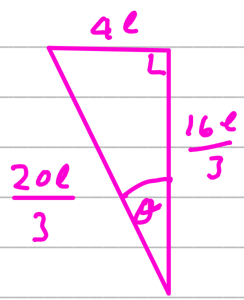
$= \dfrac{kmg \times 2l}{8l}$

$T = \dfrac{kmg}{4}$ (2)

From (1) and (2) $\dfrac{kmg}{4} = \dfrac{5mg}{6}$

$\Rightarrow k = \dfrac{20}{6} = \dfrac{10}{3}$

b)



$\Rightarrow \cos\theta = \dfrac{\frac{16l}{3}}{\frac{20l}{3}} = \dfrac{4}{5}$

$T = \dfrac{\lambda x}{8l} = \dfrac{\frac{10}{3} mg \times \left(\frac{40l}{3} - 8l\right)}{8l}$

$T = \dfrac{\frac{10}{3} mg \times \frac{16}{3} l}{8l} = \dfrac{20mg}{9}$ N

Resultant force upwards $= 2T\cos\theta - mg$

N2L $2 \times \dfrac{20mg}{9} \times \dfrac{4}{5} - mg = ma$

$ma = \left(\frac{32}{9} - 1\right)mg \Rightarrow a = \dfrac{23g}{9}$ ms^{-2}

Question 4 continued

c) Extension $\frac{40}{3}l - 8l = \frac{16l}{3} = x$

Elastic energy stored $= \frac{\lambda x^2}{16l} = \frac{\frac{10}{3}mg \times \frac{256}{9}l^2}{16l}$

$= \frac{160\,mgl}{27}$ J

Elastic energy used = gain in GPE + gain in KE

$$\frac{160\,mgl}{27} = mg\sqrt{\left(\frac{20}{3}l\right)^2 - (4l)^2} + \frac{1}{2}mv^2$$

$$\frac{160\,mgl}{27} - \frac{16}{3}mgl = \frac{1}{2}mv^2$$

$$\frac{16}{27}mgl = \frac{1}{2}mv^2$$

$$\frac{32}{27}gl = v^2$$

$$v = \sqrt{\frac{32gl}{27}} \text{ m s}^{-1}$$

d) Modulus of elasticity may be inaccurate

Question 4 continued

(Total for Question 4 is 11 marks)

5. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

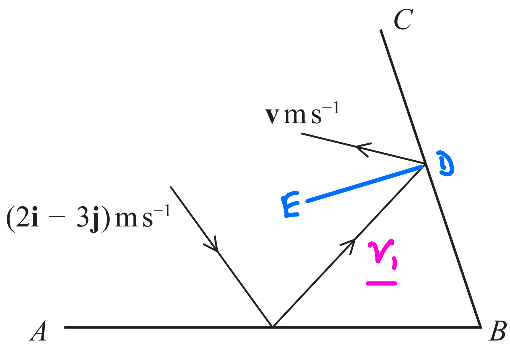


Figure 3

Figure 3 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls. The direction of $\overrightarrow{AB}$ is in the direction of the vector $\mathbf{i}$ and the direction of $\overrightarrow{BC}$ is in the direction of the vector $(-\mathbf{i} + 3\mathbf{j})$.

A small ball is projected along the floor towards wall AB so that, immediately before hitting wall AB, the velocity of the ball is $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$.

The ball hits wall AB and then hits wall BC.

The coefficient of restitution between the ball and wall AB is $\frac{1}{2}$

The coefficient of restitution between the ball and wall BC is $\frac{1}{3}$

The velocity of the ball immediately after hitting wall BC is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

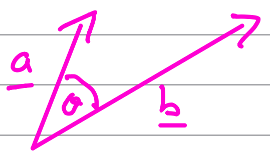
Show that $\mathbf{v} = \left(-\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$.

(12)

Let velocity after impact with AB be $\underline{V_1}$

$$\underline{V_1} = \begin{pmatrix} 2 \\ 3 \times \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

Aside



Component of $\underline{a}$ in direction of $\underline{b}$ given by $|\underline{a}|\cos\theta$

but $\cos\theta = \dfrac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$

$\therefore$ component $= \dfrac{\underline{a}.\underline{b}}{|\underline{b}|}$

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Question 5 continued

$\overrightarrow{BC} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

Let $\overrightarrow{DE}$ be a vector $\perp$ to $\overrightarrow{BC}$ at point of impact

then $\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ say

Component of $\underline{v}_1$ in direction $\overrightarrow{BC}$ both before and after impact $= \dfrac{\underline{v}_1 . \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 3 \end{pmatrix}\right|} = \dfrac{\begin{pmatrix} 2 \\ 1.5 \end{pmatrix} . \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ 3 \end{pmatrix}\right|}$

$= \dfrac{-2 + 4.5}{\sqrt{10}} = \dfrac{2.5}{\sqrt{10}}$

Component of $\underline{v}_1$ in direction $\overrightarrow{DE}$ after impact

$= \dfrac{\frac{1}{3}\begin{pmatrix} -2 \\ -1.5 \end{pmatrix} . \begin{pmatrix} -3 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} -3 \\ -1 \end{pmatrix}\right|} = \dfrac{\frac{1}{3}(6 + 1.5)}{\sqrt{10}} = \dfrac{2.5}{\sqrt{10}}$

$\underline{v} = \dfrac{\frac{2.5}{\sqrt{10}}\begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\sqrt{10}} + \dfrac{\frac{2.5}{\sqrt{10}}\begin{pmatrix} -3 \\ -1 \end{pmatrix}}{\sqrt{10}}$

$\underline{v} = 0.25\begin{pmatrix} -1 \\ 3 \end{pmatrix} + 0.25\begin{pmatrix} -3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{1}{2} \end{pmatrix}$

$\underline{v} = -\underline{i} + \frac{1}{2}\underline{j}$

S 6 1 2 9 8 A 0 1 3 2 4

Turn over

6. A particle, P, of mass $4m$ is moving along a straight line on a smooth horizontal plane.

A particle, Q, of mass $3m$ is at rest on the plane on the same straight line.

Particle P collides directly with particle Q.

Immediately before the collision the speed of P is ku, where k is a constant.

Immediately after the collision the speed of P is u and the speed of Q is $\frac{3u}{2}$

The coefficient of restitution between P and Q is e.

(a) (i) Show that there is only one possible value of k.

(ii) State the value of k and the value of e.

(11)

(b) Find the total kinetic energy lost in the collision between P and Q.

(3)

a) $4m$ $\quad$ $3m$

Possibility 1 $\quad$ P $\quad$ Q

$\xrightarrow{ku}$ $\quad$ $\xrightarrow{u}$ $\quad$ $\xrightarrow{\frac{3u}{2}}$

PCLM $\quad$ $4mku = 4mu + 3m \times \frac{3u}{2}$

$$\Rightarrow \quad 4k = 4 + \frac{9}{2}$$

$$8k = 17$$

$$k = \frac{17}{8}$$

$4m$ $\quad$ $3m$

Possibility 2 $\quad$ P $\quad$ Q

$\xrightarrow{ku}$ $\quad$ $\xleftarrow{u}$ $\quad$ $\xrightarrow{\frac{3u}{2}}$

Question 6 continued

PCLM $\quad 4mku = 3m \times \frac{3u}{2} - 4mu$

$4k = \frac{9}{2} - 4$

$4k = \frac{1}{2}$

$k = \frac{1}{8}$

$\Rightarrow \quad e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{u + \frac{3u}{2}}{\frac{1}{8}u}$

$= 20$

Impossible for $e > 1$ so $k \neq \frac{1}{8}$

and only value is $k = \frac{17}{8}$

$e = \frac{\frac{3u}{2} - u}{\frac{17}{8}u} = \frac{4}{17}$

b) Loss in KE

$\frac{1}{2}(4m)\left(\frac{17}{8}u\right)^2 - \left[\frac{1}{2}(4m)u^2 + \frac{1}{2}(3m)\left(\frac{3u}{2}\right)^2\right]$

$= \frac{289}{32}mu^2 - \frac{43}{8}mu^2 = \frac{117}{32}mu^2 \quad$ J

7.

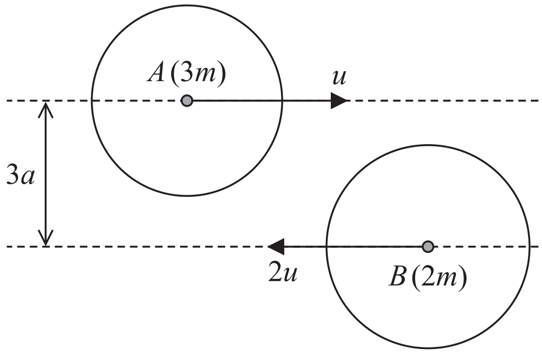


Figure 4

Two smooth uniform spheres, A and B, are moving with speeds u and $2u$ respectively on a smooth horizontal surface.

Sphere A has mass $3m$ and radius $2a$. Sphere B has mass $2m$ and radius $2a$.

The centres of the spheres are moving towards each other on parallel paths. The paths are at a distance $3a$ apart, as shown in Figure 4.

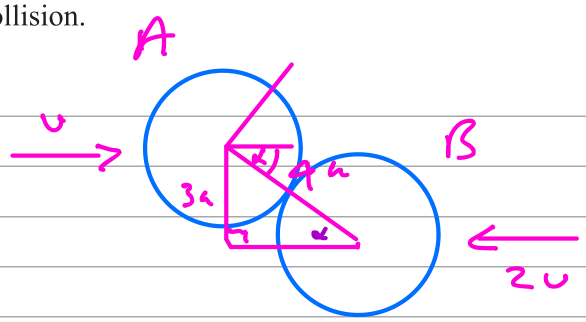
The spheres collide. The coefficient of restitution between A and B is $\frac{1}{3}$

(a) Show that the magnitude of the impulse received by A in the collision is $\frac{6\sqrt{7}}{5}mu$.
(10)

(b) Find the speed of A immediately after the collision.
(3)

(c) State how you have used the fact that the spheres are smooth when considering their collision.
(1)

a)



$\sin\alpha = \frac{3}{4}$

$\Rightarrow \cos\alpha = \frac{\sqrt{7}}{4}$

PCLM along line of centres

$3mu\cos\alpha - 2m \times 2u\cos\alpha = 3mV_A + 2mV_B$ (1)

where V_A and V_B are velocity components along line of centres after collision

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Question 7 continued

$$e = \frac{1}{3} = \frac{V_B - V_A}{3u\cos\alpha}$$

$$\Rightarrow \quad V_B - V_A = u\cos\alpha$$

$$\Rightarrow \quad V_B = V_A + u\cos\alpha \qquad (2)$$

From (1)

$$3u\cos\alpha - 4u\cos\alpha = 3V_A + 2V_B \qquad (3)$$

Sub for V_B in (3)

$$-u\cos\alpha = 3V_A + 2(V_A + u\cos\alpha)$$

$$-u\cos\alpha = 5V_A + 2u\cos\alpha$$

$$-3u\cos\alpha = 5V_A$$

$$-\frac{3u\cos\alpha}{5} = V_A \qquad (4)$$

Impulse on A = change in momentum along line of centres

(since no change in momentum $\perp$ to line of centres)

$$\text{Impulse} = 3mV_A - 3mu\cos\alpha$$

$$= -\frac{9mu\cos\alpha}{5} - 3mu\cos\alpha$$

$$= -\frac{24mu\cos\alpha}{5}$$

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Question 7 continued

$$= -\frac{24mu}{5} \times \frac{\sqrt{7}}{4}$$

$$= -\frac{6\sqrt{7}mu}{5}$$

Magnitude of impulse $= \frac{6\sqrt{7}mu}{5}$ Ns

b) Speed of A after collision

$$= \sqrt{V_A^2 + (u\sin\alpha)^2}$$

$$= \sqrt{\left(-\frac{3u}{5} \times \frac{\sqrt{7}}{4}\right)^2 + \left(u \times \frac{3}{4}\right)^2}$$

$$= \sqrt{\frac{63u^2}{400} + \frac{9u^2}{16}}$$

$$= \frac{3\sqrt{2}u}{5} \text{ ms}^{-1}$$

c) Impulse acts along line of centres

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