

- 6 The function $f(x)$ is defined by $f(x) = 1 + 2\sin x$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

domain

- (i) Show that $f^{-1}(x) = \arcsin\left(\frac{x-1}{2}\right)$ and state the domain of this function.

[4]

Fig. 6 shows a sketch of the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

Graphs of inverse functions are mirror images of each other about $y=x$

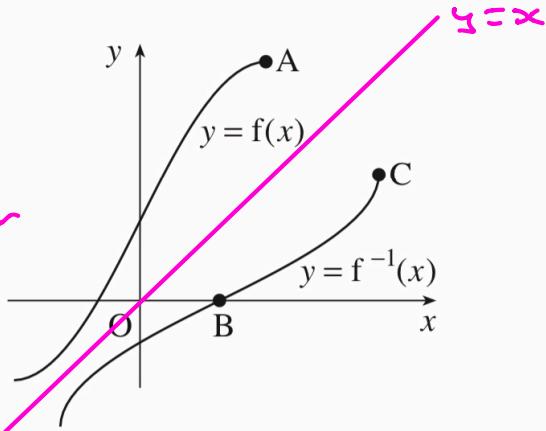


Fig. 6

- (ii) Write down the coordinates of the points A, B and C.

[3]

$$\text{i) } f(x) = 1 + 2\sin x$$

$$\text{Let } y = 1 + 2\sin x$$

$$\begin{matrix} \text{swap} \\ \text{variables} \end{matrix} \quad x = 1 + 2\sin y$$

$$x - 1 = 2\sin y$$

$$\frac{x-1}{2} = \sin y$$

$$\sin^{-1}\left(\frac{x-1}{2}\right) = y$$

$$\therefore f^{-1}(x) = \sin^{-1}\left(\frac{x-1}{2}\right)$$

$$\text{Find range of } f(x) \quad f(-\frac{\pi}{2}) = 1 + 2\sin\left(-\frac{\pi}{2}\right) = -1$$

$$f\left(\frac{\pi}{2}\right) = 1 + 2 \sin \frac{\pi}{2} = 3$$

Range $-1 \leq f(x) \leq 3$

For inverse functions the domain of one is the range of the other

\therefore domain of $f^{-1}(x)$
is given by $-1 \leq x \leq 3$

ii) $A\left(\frac{\pi}{2}, 3\right)$

$B(1, 0)$

$C\left(3, \frac{\pi}{2}\right)$

3 Fig. 3 shows the curve defined by the equation $y = \arcsin(x - 1)$, for $0 \leq x \leq 2$.

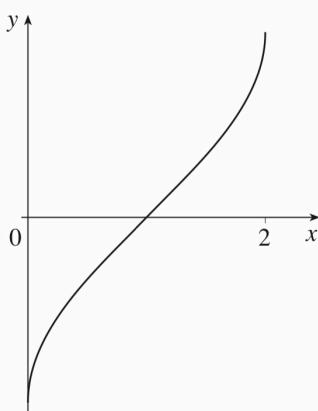


Fig. 3

- (i) Find x in terms of y , and show that $\frac{dx}{dy} = \cos y$. [3]

- (ii) Hence find the exact gradient of the curve at the point where $x = 1.5$. [4]

i)

$$y = \sin^{-1}(x-1)$$

$$\sin y = x - 1$$

$$\underline{x = 1 + \sin y}$$

$$\frac{dx}{dy} = 0 + \cos y = \cos y$$

ii)

$$\begin{aligned} \text{when } x &= 1.5, \quad y = \sin^{-1}(1.5 - 1) \\ &= \sin^{-1} \frac{1}{2} \\ &= \frac{\pi}{6} \end{aligned}$$

$$\frac{dx}{dy} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{dy}{dx} = \frac{2}{\sqrt{3}} \quad \text{or} \quad \frac{2\sqrt{3}}{3}$$

- 9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \frac{1}{\sqrt{2x - x^2}}$.

The curve has asymptotes $x = 0$ and $x = a$.

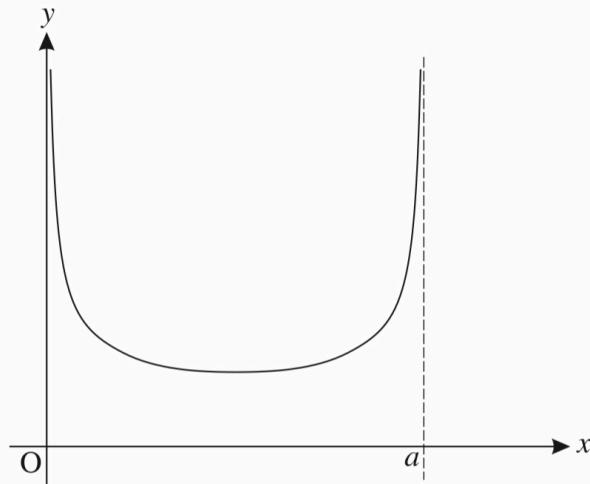


Fig. 9

- (i) Find a . Hence write down the domain of the function.

[3]

(ii) Show that $\frac{dy}{dx} = \frac{x-1}{(2x-x^2)^{\frac{3}{2}}}$.

Hence find the coordinates of the turning point of the curve, and write down the range of the function.

[8]

i) when $\sqrt{2x-x^2} = 0$
 $\sqrt{x(2-x)} = 0$
 $\Rightarrow x = 0 \text{ or } x = 2$
 $\Rightarrow \underline{a = 2}$

Domain $0 < x < a$

ii) $y = \frac{1}{\sqrt{2x-x^2}}$
 $y = (2x-x^2)^{-\frac{1}{2}}$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{2}(2x-x^2)^{-\frac{1}{2}} \times (2-2x) \\ &= (x-1)(2x-x^2)^{-\frac{1}{2}}\end{aligned}$$

At E.p. $\frac{dy}{dx} = 0 \Rightarrow x-1 = 0$

$$\underline{x = 1}$$

when $x = 1 \quad f(1) = \frac{1}{\sqrt{2(1)-1^2}} = \frac{1}{1} = 1$

E.p. $(1, 1) \therefore \text{Range } f(x) \geq 1$

- 2 The functions $f(x)$ and $g(x)$ are defined for all real numbers x by

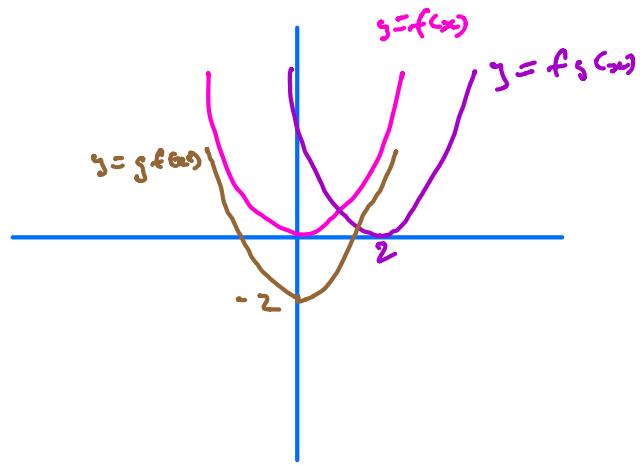
$$f(x) = x^2, \quad g(x) = x - 2.$$

- (i) Find the composite functions $fg(x)$ and $gf(x)$. [3]
- (ii) Sketch the curves $y = f(x)$, $y = fg(x)$ and $y = gf(x)$, indicating clearly which is which. [2]

i)

$$fg(x) = f(x-2) = (x-2)^2$$

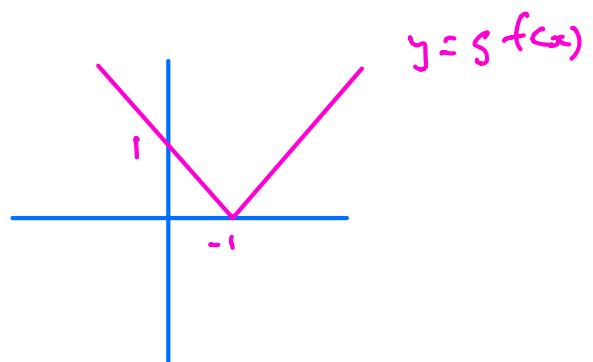
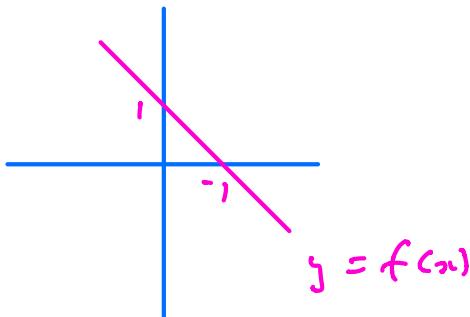
$$gf(x) = g(x^2) = x^2 - 2$$



- 2 Given that $f(x) = 1 - x$ and $g(x) = |x|$, write down the composite function $gf(x)$.

On separate diagrams, sketch the graphs of $y = f(x)$ and $y = gf(x)$. [3]

$$\begin{aligned} gf(x) &= g(1-x) \\ &= |1-x| \end{aligned}$$



- 1 Fig.1 shows the graphs of $y = |x|$ and $y = |x - 2| + 1$. The point P is the minimum point of $y = |x - 2| + 1$, and Q is the point of intersection of the two graphs.

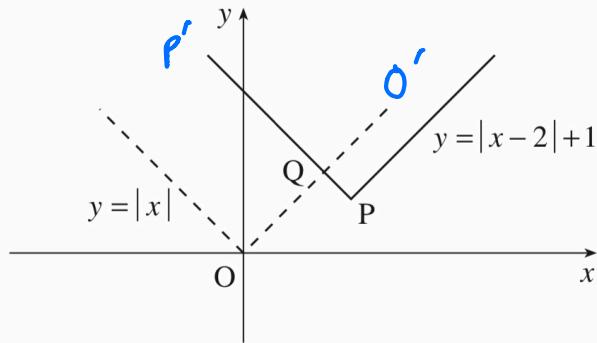


Fig. 1

- (i) Write down the coordinates of P. [1]
- (ii) Verify that the y-coordinate of Q is $1\frac{1}{2}$. [4]

i) $P(2, 1)$

ii) when $y = 1.5$ $1.5 = |x - 2| + 1$
 $\frac{1}{2} = |x - 2|$
 $\Rightarrow x = 1.5 \text{ or } 2.5$

when $y = 1.5$ $1.5 = |x|$
 $\Rightarrow x = 1.5 \text{ or } -1.5$

\therefore Q point of intersection is $(1.5, 1.5)$

Alternatively PP' is line $y = -x + 2 + 1$
 $y = -x + 3$

OO' is line $y = x$

$$\text{Solve } \begin{cases} y = x \\ y = -x + 3 \end{cases}$$

Sub for x

$$\begin{aligned} y &= -y + 3 \\ 2y &= 3 \\ y &= \frac{3}{2} \quad \text{at point of intersection} \end{aligned}$$

Show $x^3 - 3x + 1 = 0$

has a solution between $x = 1$ and $x = 2$

$$\begin{aligned} x = 1 &\quad x^3 - 3x + 1 \\ &= 1^3 - 3(1) + 1 \\ &= -1 \end{aligned}$$

$$\begin{aligned} x = 2 &\quad x^3 - 3x + 1 \\ &= 2^3 - 3(2) + 1 \\ &= 8 - 6 + 1 \\ &= 3 \end{aligned}$$

Sign change from $-ve$ to $+ve$
on continuous function, \therefore root between
 $x = 1$ and $x = 2$
