Further Mathematics

Advanced Subsidiary

Paper 1: Core Pure Mathematics

Paper 1 Core Pure Mathematics	
You must have: Mathematical Formulae and Statistical Tables, calculator	
Time	1 hour 40 minutes

Name	
Class	
Teacher name	

Total marks	/80

1 The complex numbers z_1 and z_2 are given by $z_1 = 5 - 2\text{i}$ and $z_2 = 1 + 3\text{i}$.

a Express $\frac{z_1}{z_2}$ in the form $a + b\text{i}$, where a and b are real.

(3)

b Find $|z_2^2|$.

(2)

(Total for Question 1 is 5 marks)

2

$$\mathbf{M} = \begin{pmatrix} p & -4 \\ 1 & p+1 \end{pmatrix}$$

where $p > 0$ is a real constant.

a Find det**M** in terms of p.

(2)

b Show that **M** is non-singular for all values of p.

(2)

c Given that $\mathbf{M} + 10\mathbf{M}^{-1} = 5\mathbf{I}$ where **I** is the 2×2 identity matrix, find the value of p.

(3)

(Total for Question 2 is 7 marks)

3

$$g(z) = z^3 + pz^2 + 14z - 20$$

Given g(1 – 3i) = 0,

a find the value of p

(3)

b find the other two roots of of g(z) = 0.

(3)

(Total for Question 3 is 6 marks)

4 A student makes the following claim:

'If P is a reflection in the line $y = x$ and Q is a rotation 90° anticlockwise about the origin then P followed by Q is the same as a reflection in the y-axis.'

Show, using matrix multiplication, that the student is correct.

(3)

(Total for Question 4 is 3 marks)

5 The points A, B and C have coordinates $A(3, -2, 4)$, $B(-2, 7, 1)$ and $C(4, -2, -1)$.

a Find $\overrightarrow{AB}.\overrightarrow{AC}$.

(2)

b Hence, or otherwise, find the area of triangle ABC to 3 significant figures.

(5)

(Total for Question 5 is 7 marks)

6 a Show that

$$\sum_{r=1}^{n} r^2(2r-3) = \tfrac{1}{2}n(n+1)(n^2-n-1)$$

(4)

b Find the value of n such that

$$\sum_{r=1}^{n} r^2(2r-3) = \sum_{r=1}^{n} 29r$$

(4)

(Total for Question 6 is 8 marks)

7 Figure 1 shows the central cross-section, *OPQRS*, of a circular football trophy display, which is made of solid metal. Measurements of the height and diameter of the display have been taken in order to calculate the amount of metal needed to create it.

Using these measurements, the cross-sectional curve *QR*, shown in Figure 2, is modelled as a curve with equation $y = ax^2 + 20$, $-10 \leqslant x \leqslant 10$, where a is a constant and O is the fixed origin.

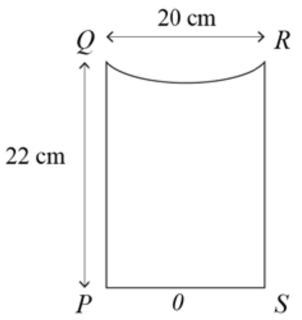

Figure 1

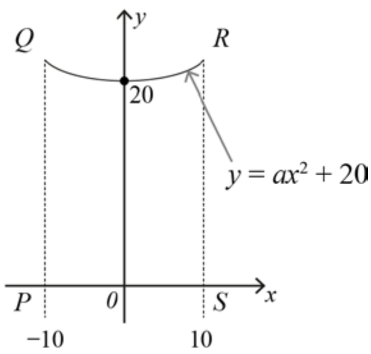

Figure 2

a Calculate the value of a.

(1)

b Find the volume of metal that was required to make the display according to this model.

Give your answer as an exact value in terms of π.

(6)

c State a limitation of the model.

(1)

(Total for Question 7 is 8 marks)

8 The region R in an Argand diagram is satisfied by the inequalities

$$|z + 1 + 7\text{i}| \leqslant |z - 11 - 5\text{i}| \quad \text{and} \quad |z - 10 + 6\text{i}| \leqslant 5$$

a Draw an Argand diagram and shade in the region R.

(6)

b Given that

$$\arg(z - 5 - 4\text{i}) = \theta \quad \text{and} \quad |z - 10 + 6\text{i}| = 5$$

have exactly one solution, find the two possible values of θ correct to two decimal places.

(3)

(Total for Question 8 is 9 marks)

9 The cubic equation

$$x^3 - 4x^2 - 6x + 3 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$ giving your answer in the form,

$$w^3 + pw^2 + qw + r = 0$$

where p, q and r are integers to be found.

(5)

(Total for Question 9 is 5 marks)

10 At the beginning of 2015, there were 4000 people living in a village.

The people were classified as either adult males, adult females or children.

Initially, there were 200 more adult females than adult males.

After one year,

- the adult male population had increased by 3.5%
- the adult female population had increased by 2.5%
- the number of children has decreased by 3.5%
- the total population had increased by 80.

Form and solve a matrix equation to calculate the number of adult males, adult females and children at the beginning of 2015.

(7)

(Total for Question 10 is 7 marks)

11 Prove by induction that, for all positive integers, $5^{n+1} + 6^{2n-1}$ is divisible by 31.

(7)

(Total for Question 11 is 7 marks)

12 A pilot is planning to fly an aeroplane in a straight line from point $A(-3, 1, 7)$ to point $B(-8, 6, 5)$, where the unit of distance is kilometres.

A 'no-fly' zone exists surrounding a point located at the origin, so that the aeroplane cannot fly within eight kilometres of point O.

a Use the model to determine whether or not the plane is able to fly in a straight line from point A to point B without entering the no-fly zone.

(7)

b Describe a limitation of the model.

(1)

(Total for Question 12 is 8 marks)

TOTAL FOR PAPER IS 80 MARKS

