

## Mixed Exercise 13

$$\begin{aligned}
 1b) \quad & \int \left( x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx \\
 &= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C \\
 &= \frac{3}{4} x^{\frac{4}{3}} + \frac{3}{2} x^{\frac{2}{3}} + C
 \end{aligned}$$


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$$\begin{aligned}
 3b) \quad & \int (5x + 2) x^{\frac{1}{2}} dx \\
 &= \int (5x^{\frac{3}{2}} + 2x^{\frac{1}{2}}) dx \\
 &= \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + C \\
 &= 2x^{\frac{5}{2}} + \frac{4}{3} x^{\frac{3}{2}} + C
 \end{aligned}$$


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$$5) \quad \frac{dx}{dt} = (t+1)^2 = t^2 + 2t + 1$$

$$x = \int \frac{dx}{dt} dt = \int (t^2 + 2t + 1) dt$$

$$x = \frac{t^3}{3} + t^2 + t + C$$

$$\begin{cases} x=0 \\ t=2 \end{cases}$$

$$0 = \frac{2^3}{3} + 2^2 + 2 + C$$

$$0 = \frac{8}{3} + 4 + 2 + C$$

$$-\frac{26}{3} = C$$

$$x = \frac{t^3}{3} + t^2 + t - \frac{26}{3}$$

when  $t=3$   $x = \frac{3^3}{3} + 3^2 + 3 - \frac{26}{3}$

$$x = 9 + 9 + 3 - \frac{26}{3}$$

$$x = \frac{37}{3}$$

7)  $y^{\frac{1}{2}} = 3x^{\frac{1}{4}} - 4x^{-\frac{1}{4}}$

$$y = \left(3x^{\frac{1}{4}} - 4x^{-\frac{1}{4}}\right)^2$$

$$y = 9x^{\frac{1}{2}} - 12 + 16x^{-\frac{1}{2}}$$

a)  $\frac{dy}{dx} = \frac{9}{2}x^{-\frac{1}{2}} - 8x^{-\frac{3}{2}}$

b)  $\int y dx = \int \left(9x^{\frac{1}{2}} - 12 + 16x^{-\frac{1}{2}}\right) dx$

$$= \frac{9x^{\frac{3}{2}}}{\frac{3}{2}} - 12x + \frac{16x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 6x^{\frac{3}{2}} - 12x + 32x^{\frac{1}{2}} + c$$

$$\begin{aligned}
 1 \text{ a) } & \int (x+1)(2x-5) dx \\
 & = \int (2x^2 + 2x - 5x - 5) dx \\
 & = \int (2x^2 - 3x - 5) dx \\
 & = \frac{2x^3}{3} - \frac{3x^2}{2} - 5x + C
 \end{aligned}$$


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$$2) \quad f'(x) = x^2 - 3x - \frac{2}{x^2}$$

$$f'(x) = x^2 - 3x - 2x^{-2}$$

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - \frac{2x^{-1}}{-1} + C$$

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + \frac{2}{x} + C$$

$$(1,1) \quad f(1) = 1 = \frac{1^3}{3} - \frac{3(1)^2}{2} + \frac{2}{1} + C$$

$$1 = \frac{1}{3} - \frac{3}{2} + 2 + C$$

$$1 - \frac{1}{3} + \frac{3}{2} - 2 = C$$

$$\frac{1}{6} = C$$

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + \frac{2}{x} + \frac{1}{6}$$