Exercise 7B Q16

$$f(x) = 3x^3 - 14x^2 - 47x - 14$$

Find remainder when dividing by $(x-3)$

$$
\begin{array}{r|l}
 & 3x^2 - 5x - 62 \\
\hline
x-3 & 3x^3 - 14x^2 - 47x - 14 \\
 & 3x^3 - 9x^2 \\
\hline
 & \quad -5x^2 - 47x \\
 & \quad -5x^2 + 15x \\
\hline
 & \qquad -62x - 14 \\
 & \qquad -62x + 186 \\
\hline
 & \qquad\qquad -200
\end{array}
$$

remainder $= -200$

$$f(x) = (x-3)(3x^2 - 5x - 62) - 200$$

Consider $f(3) = 3(3)^3 - 14(3)^2 - 47(3) - 14$

$= 81 - 126 - 141 - 14$

$= -200$

This is an example of the remainder theorem

Not on syllabus but

If a polynomial $f(x)$ is divided by $(x-a)$

then the remainder is given by $f(a)$

The factor theorem (which is on syllabus) is a special case of the remainder theorem

$(x-a)$ is factor of $f(x)$

if and only $f(a)=0$

Using the factor theorem

Fully factorise $f(x) = x^3 + 6x^2 + 5x - 12$

$f(1) = 1^3 + 6(1)^2 + 5(1) - 12 = 0$ ✓

$\therefore (x-1)$ is a factor

$f(2) = 2^3 + 6(2)^2 + 5(2) - 12 = 30 \neq 0$ ✗

$f(-2) = (-2)^3 + 6(-2)^2 + 5(-2) - 12 = -6 \neq 0$ ✗

$f(3) = 3^3 + 6(3)^2 + 5(3) - 12 \neq 0$ ✗

$f(-3) = (-3)^3 + 6(-3)^2 + 5(-3) - 12$

$= -27 + 54 - 15 - 12 = 0$ ✓

$(x+3)$ is a factor

$\therefore \quad f(x) = (x-1)(x+3)(x+4)$

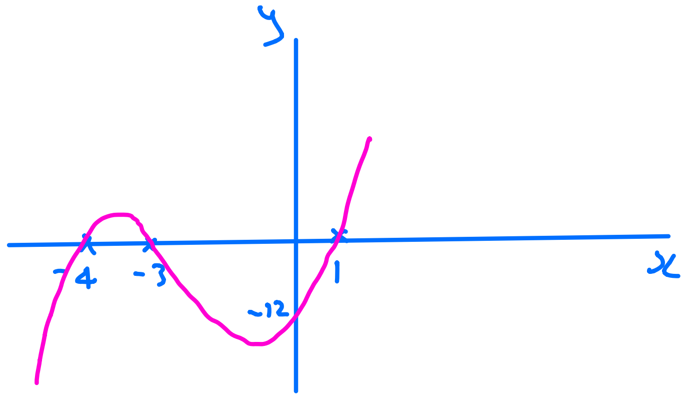
Check $f(-4) = (-4)^3 + 6(-4)^2 + 5(-4) - 12$

$= -64 + 96 - 20 - 12 = 0$ ✓

Answer $f(x) = (x-1)(x+3)(x+4)$

Sketch Graph

$y = f(x)$



Exercise 7C

4) Show $(x-5)$ is a factor of $x^3 - 7x^2 + 2x + 40$

Let $f(x) = x^3 - 7x^2 + 2x + 40$

$$f(5) = 5^3 - 7(5)^2 + 2(5) + 40$$
$$= 125 - 175 + 10 + 40$$
$$= 0$$

By factor theorem $f(5) = 0 \Rightarrow (x-5)$ is a factor

$$\begin{array}{r|l} & x^2 - 2x - 8 \\ \hline x-5 & x^3 - 7x^2 + 2x + 40 \\ & \underline{x^3 - 5x^2} \\ & \quad -2x^2 + 2x \\ & \quad \underline{-2x^2 + 10x} \\ & \qquad -8x + 40 \\ & \qquad \underline{-8x + 40} \\ & \qquad\qquad 0 \end{array}$$

$$f(x) = (x-5)(x^2 - 2x - 8)$$
$$= (x-5)(x-4)(x+2)$$

13)

$$f(x) = 3x^3 - 12x^2 + 6x - 24$$

$$f(4) = 3(4)^3 - 12(4)^2 + 6(4) - 24$$

$$= 192 - 192 + 24 - 24$$

$$= 0$$

By factor theorem $f(4) = 0 \Rightarrow (x-4)$ is a factor

$$\begin{array}{r|l} & 3x^2 + 6 \\ \hline x - 4 & 3x^3 - 12x^2 + 6x - 24 \\ & \underline{3x^3 - 12x^2} \\ & \qquad\qquad +6x - 24 \\ & \qquad\qquad \underline{+6x - 24} \end{array}$$

$$f(x) = (x-4)(3x^2+6)$$

$3x^2 + 6 > 0$ for all x

$\therefore$ $x = 4$ is only root

Homework Q13, Q14, Q15 Exercise 7B