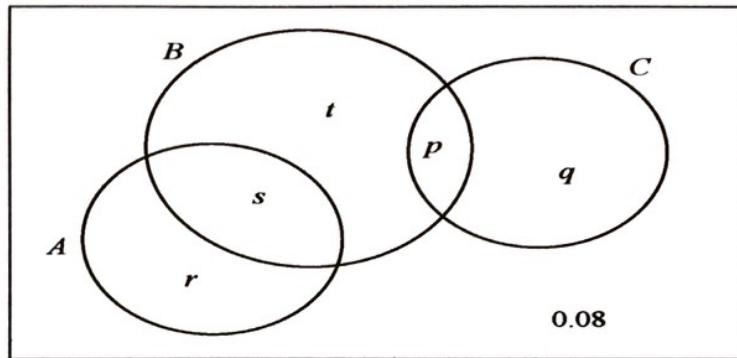


Venn Diagrams

3. The Venn diagram shows three events A , B and C , where p , q , r , s and t are probabilities.



$P(A) = 0.5$, $P(B) = 0.6$ and $P(C) = 0.25$ and the events B and C are independent.

- (a) Find the value of p and the value of q . (2)
- (b) Find the value of r . (2)
- (c) Hence write down the value of s and the value of t . (2)
- (d) State, giving a reason, whether or not the events A and B are independent. (2)
- (e) Find $P(B | A \cup C)$. (3)

a) B, C independent

$$\therefore P(B \cap C) = P(B) \times P(C)$$

$$p = 0.6 \times 0.25$$

$$\underline{p = 0.15}$$

$$q = P(C) - p = 0.25 - 0.15$$

$$\underline{q = 0.1}$$

b)

$$r = 1 - 0.08 - q - P(B)$$

$$r = 1 - 0.08 - 0.1 - 0.6$$

$$\underline{r = 0.22}$$

c)

$$\begin{aligned} s &= P(A) - r \\ s &= 0.5 - 0.22 \\ \underline{s = 0.28} \end{aligned}$$

$$\begin{aligned} t &= P(B) - p - s \\ t &= 0.6 - 0.15 - 0.28 \\ \underline{t = 0.17} \end{aligned}$$

d) $P(A \cap B) = s = 0.28$

$$P(A) \times P(B) = 0.5 \times 0.6 = 0.3$$

$$0.3 \neq 0.28$$

So A and B are not independent

e) $P(B | A \cup C) = \frac{P(B \cap (A \cup C))}{P(A \cup C)}$

$$= \frac{p + s}{0.5 + 0.25}$$

$$= \frac{0.15 + 0.28}{0.75}$$

$$= \frac{43}{75}$$

The Normal Distribution

$$i) X \sim N(\mu, \sigma^2)$$

$$Z \sim N(0, 1^2)$$

$$Z = \frac{x - \mu}{\sigma}$$

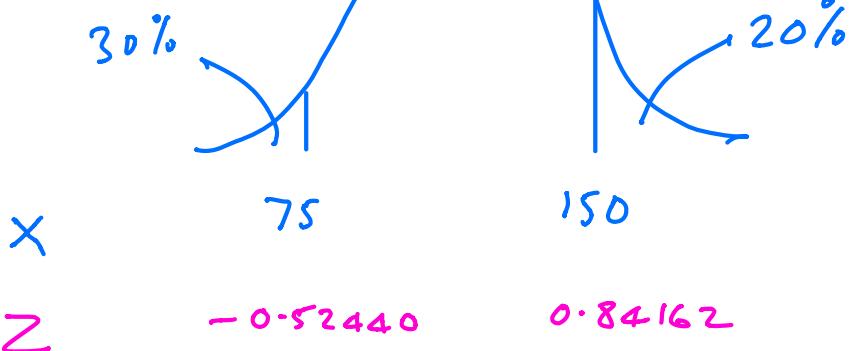
Rearranging

$$\sigma z = x - \mu$$

$$\sigma z + \mu = x$$

$$\Phi^{-1}(80\%) = 0.84162$$

$$\Phi^{-1}(30\%) = -0.52440$$



$$-0.52440 \sigma + \mu = 75$$

$$0.84162 \sigma + \mu = 150$$

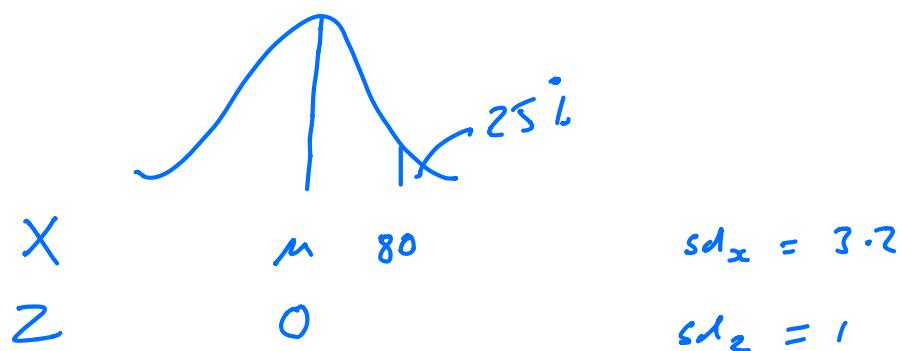
By calc $\sigma = 54.90$
 $\mu = 103.79$

$$X \sim N(103.79, 54.9^2)$$

2) $X \sim N(\mu, 3.2^2)$

Given that $P(X > 80) = 0.25$

Find μ



$$\Phi^{-1}(0.75) = 0.67449$$

$$Z = \frac{x - \mu}{\sigma}$$

$$Z \sigma = x - \mu$$

$$\mu = x - Z \sigma$$

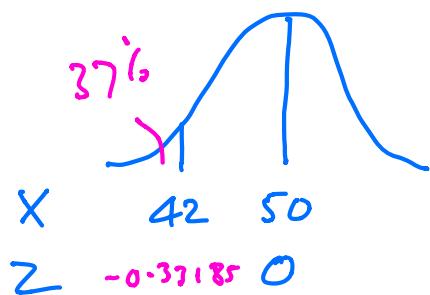
$$\mu = 80 - 0.67449 \times 3.2$$

$$\mu = 77.84$$

3)

$$X \sim N(50, \sigma^2)$$

Given $P(X > 42) = 63\%$ find σ



$$\Phi^{-1}(0.37) = -0.37185$$

$$Z = \frac{x - \mu}{\sigma}$$

$$\sigma z = x - \mu$$

$$\sigma = \frac{x - \mu}{z} = \frac{42 - 50}{-0.37185}$$

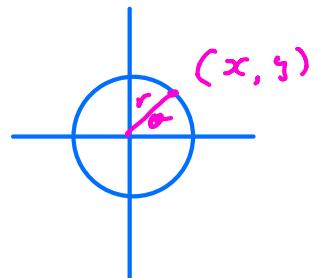
$$\sigma = 24.1$$

Integration with Parametric Eqns

Circle radius r centre origin

$$x = r \cos \theta$$

$$y = r \sin \theta$$

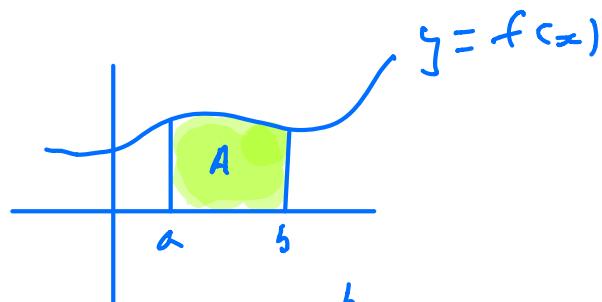


$$x^2 + y^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$$

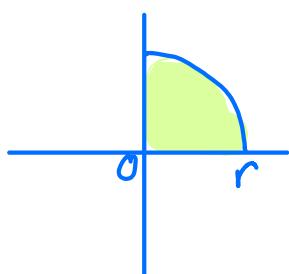
$$x^2 + y^2 = r^2 (\cos^2 \theta + \sin^2 \theta)$$

$$\underline{x^2 + y^2 = r^2}$$

In general



$$\text{Area } A = \int_a^b y \, dx$$



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$= \int_0^r y \, dx$$

$$= \int y \frac{dx}{d\theta} d\theta$$

$$= \int_{\frac{\pi}{2}}^0 r \sin \theta x (-r \sin \theta) d\theta$$

$$x = 0, r \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

$$x = r \quad r \cos \theta = r \Rightarrow \theta = 0$$

$$= r^2 \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$2\sin^2 \theta = 1 - \cos 2\theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$= r^2 \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \frac{r^2}{2} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}}$$

$$= \frac{r^2}{2} \left[\frac{\pi}{2} - \frac{1}{2} \sin \pi - (0 - \frac{1}{2} \sin 0) \right]$$

$$= \frac{r^2}{2} \times \left[\frac{\pi}{2} - 0 - 0 + 0 \right]$$

$$= \frac{\pi r^2}{4}$$