Differences Homework

1 • Use the result $\frac{1}{5r-1} - \frac{1}{5r+4} \equiv \frac{5}{(5r-1)(5r+4)}$ and the method of differences to find

$$\sum_{r=1}^{n} \frac{1}{(5r-1)(5r+4)},$$

simplifying your answer. **[6]**

2 • Given that $\frac{3}{(3r-1)(3r+2)} \equiv \frac{1}{3r-1} - \frac{1}{3r+2}$, find $\sum_{r=1}^{20} \frac{1}{(3r-1)(3r+2)}$, giving your answer as an exact fraction. **[5]**

3 • **(i)** Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. **[2]**

(ii) Use the method of differences to find $\sum_{r=1}^{30} \frac{1}{(2r+1)(2r+3)}$, expressing your answer as a fraction. **[5]**

4 • You are given that $\frac{3}{(5+3x)(2+3x)} \equiv \frac{1}{2+3x} - \frac{1}{5+3x}$.

(i) Use this result to find $\sum_{r=1}^{100} \frac{1}{(5+3r)(2+3r)}$, giving your answer as an exact fraction. **[5]**

(ii) Write down the limit to which $\sum_{r=1}^{n} \frac{1}{(5+3r)(2+3r)}$ converges as n tends to infinity. **[1]**