

Exercise 11F

$$4g) \int_0^{\frac{\pi}{3}} \sin x \ln(\sec x) dx = \frac{d}{dx} \left(\frac{1}{\cos x} \right)^{-1} - 1 (\cos x)^{-2} (-\sin x)$$

$$\text{Let } u = \ln(\sec x)$$

$$\Rightarrow \frac{du}{dx} = \frac{\sin x}{\cos^2 x} = \tan x$$

$$\text{Let } \frac{dv}{dx} = \sin x$$

$$\Rightarrow v = -\cos x$$

$$\int u \frac{dv}{dx} dx = uv - v \frac{du}{dx}$$

$$\begin{aligned} \int_0^{\frac{\pi}{3}} \sin x \ln(\sec x) dx &= \left[-\cos x \ln|\sec x| \right]_0^{\frac{\pi}{3}} + \int_0^{\frac{\pi}{3}} \cos x \tan x dx \\ &= \left[-\cos x \ln|\sec x| \right]_0^{\frac{\pi}{3}} + \int_0^{\frac{\pi}{3}} \sin x dx \end{aligned}$$

$$\text{Solution} = \left[-\cos x \ln|\sec x| \right]_0^{\frac{\pi}{3}} + \left[-\cos x \right]_0^{\frac{\pi}{3}}$$

$$\begin{aligned}&= \left(-\cos \frac{\pi}{3} \ln \left| \frac{1}{\cos \frac{\pi}{3}} \right| \right) - \left(-\cos 0 \ln \left| \frac{1}{\cos 0} \right| \right) + \left(-\cos \frac{\pi}{3} \right) - \left(-\cos 0 \right) \\&= -\frac{1}{2} \ln 2 - 0 - \frac{1}{2} + 1 \\&= \frac{1}{2} - \frac{1}{2} \ln 2\end{aligned}$$

$$6) \text{ a) } \int \sqrt{8-x} dx$$

Let $v = 8-x$

$$= \int -v^{\frac{1}{2}} du$$

$$= -\frac{v^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= -\frac{2}{3} v^{\frac{3}{2}} + C$$

$$= -\frac{2}{3} (8-x)^{\frac{3}{2}} + C$$

$$5) \int (x-2)\sqrt{8-x} dx$$

Let $v = x-2$ Let $\frac{dv}{dx} = \sqrt{8-x}$

$$\Rightarrow \frac{dv}{dx} = 1 \quad \Rightarrow v = -\frac{2}{3}(8-x)^{\frac{3}{2}}$$

$$\int v \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int (x-2)\sqrt{8-x} dx = -\frac{2}{3}(x-2)(8-x)^{\frac{3}{2}} - \int -\frac{2}{3}(8-x)^{\frac{3}{2}} dx$$

$$= -\frac{2}{3}(x-2)(8-x)^{\frac{3}{2}} + \frac{2}{3} \int (8-x)^{\frac{3}{2}} dx$$

Aside

$$\begin{aligned}
 & \int (8-x)^{\frac{3}{2}} dx \quad \text{Let } u = 8-x \\
 &= \int -u^{\frac{3}{2}} du \quad \frac{du}{dx} = -1 \\
 &= -\frac{u^{\frac{5}{2}}}{\frac{5}{2}} = -\frac{2}{5} u^{\frac{5}{2}} = -\frac{2}{5} (8-x)^{\frac{5}{2}}
 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{2}{3}(x-2)(8-x)^{\frac{3}{2}} + \frac{2}{3}\left(-\frac{2}{5}\right)(8-x)^{\frac{5}{2}} + C \\
 &= -\frac{2}{3}(x-2)(8-x)^{\frac{3}{2}} - \frac{4}{15}(8-x)^{\frac{5}{2}} + C \\
 &= (8-x)^{\frac{3}{2}} \left[-\frac{2}{3}(x-2) - \frac{4}{15}(8-x) \right] + C \\
 &= (8-x)^{\frac{3}{2}} \left[-\frac{10}{15}x + \frac{20}{15} - \frac{32}{15} + \frac{4}{15}x \right] + C \\
 &= (8-x)^{\frac{3}{2}} \left[-\frac{6}{15}x - \frac{12}{15} \right] + C \\
 &= (8-x)^{\frac{3}{2}} \left[-\frac{2}{5}x - \frac{4}{5} \right] + C \\
 &= -\frac{2}{5}(8-x)^{\frac{3}{2}}(x+2) + C
 \end{aligned}$$

c)

$$\int_4^7 (x-2)\sqrt{8-x} dx$$

$$= \left[-\frac{2}{5} (8-x)^{\frac{3}{2}} (x+2) \right]_4^7$$

$$- \frac{2}{5} \left[(8-7)^{\frac{3}{2}} (7+2) - (8-4)^{\frac{3}{2}} (4+2) \right]$$

$$- \frac{2}{5} \left[9 - 48 \right]$$

$$= - \frac{2}{5} (-39) = + \frac{78}{5}$$

(b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{(5-x)} dx \quad (4)$$

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)} dx$.

(2)
(Total 8 marks)

$$\text{Let } v = x-1$$

$$\frac{dv}{dx} = 1$$

$$\text{Let } \frac{dv}{dx} = (5-x)^{\frac{1}{2}}$$

$$\Rightarrow v = - \frac{(5-x)^{\frac{3}{2}}}{\frac{3}{2}}$$

$$v = - \frac{2}{3} (5-x)^{\frac{3}{2}}$$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int (x-1) \sqrt{5-x} dx = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3} \int (5-x)^{\frac{5}{2}} dx$$

$$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{2}{5} \cdot \frac{2}{3} (5-x)^{\frac{5}{2}} + C$$

$$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15} (5-x)^{\frac{5}{2}} + C$$

$$= (5-x)^{\frac{3}{2}} \left[-\frac{2}{3}x + \frac{2}{3} - \frac{4}{15}(5-x) \right] + C$$

$$= (5-x)^{\frac{3}{2}} \left[-\frac{2}{3}x + \frac{2}{3} - \frac{20}{15} + \frac{4}{15}x \right] + C$$

$$= (5-x)^{\frac{3}{2}} \left[-\frac{6}{15}x - \frac{10}{15} \right] + C$$

$$= -\frac{2}{15}(5-x)^{\frac{3}{2}} \left[3x + 5 \right] + C$$

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)} dx$.

$$= -\frac{2}{15} \left[(5-5)^{\frac{3}{2}} (15-5) - 4^{\frac{3}{2}} \times 8 \right]$$

$$= -\frac{2}{15} [0 - 64] = + \frac{128}{15}$$