Binomial Distribution

$X \sim B(\overset{n}{20}, \overset{p}{0.55})$

$X = 16$

$P(X \geq 16)$

$= 1 - P(X \leq 15)$

$1 - 0.9811$

$= 0.0189 < 5\%$

Using Calculator

$X \sim B(\overset{n}{27}, \overset{p}{0.44})$

1) Find $P(X = 13) = 0.1386$

2) Find $P(X > 10) = 1 - P(X \leq 10)$

$= 1 - 0.2987$

$= 0.7013$

3) Find $P(X < 15)$

$= P(X \leq 14) = 0.8451$

4) Find $P(9 \leq X \leq 12)$

$= P(X \leq 12) - P(X \leq 8)$

$= 0.5975 - 0.0935 = 0.5040$

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8 At a doctor's surgery, records show that 20% of patients who make an appointment fail to turn up. During afternoon surgery the doctor has time to see 16 patients.

There are 16 appointments to see the doctor one afternoon.

(i) Find the probability that all 16 patients turn up. [2]

(ii) Find the probability that more than 3 patients do not turn up. [3]

To improve efficiency, the doctor decides to make more than 16 appointments for afternoon surgery, although there will still only be enough time to see 16 patients. There must be a probability of at least 0.9 that the doctor will have enough time to see all the patients who turn up.

(iii) The doctor makes 17 appointments for afternoon surgery. Find the probability that at least one patient does not turn up. Hence show that making 17 appointments is satisfactory. [3]

(iv) Now find the greatest number of appointments the doctor can make for afternoon surgery and still have a probability of at least 0.9 of having time to see all patients who turn up. [4]

A computerised appointment system is introduced at the surgery. It is decided to test, at the 5% level, whether the proportion of patients failing to turn up for their appointments has changed. There are always 20 appointments to see the doctor at morning surgery. On a randomly chosen morning, 1 patient does not turn up.

(v) Write down suitable hypotheses and carry out the test. [7]

i) $X \sim B(\overset{n}{16}, \overset{p}{0.2})$

$P(X=16) \quad 0.8^{16} = 0.0281$

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ii) $P(X > 3) = 1 - P(X \leq 3)$

$= 1 - 0.5981$

$= 0.4019$

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iii) $X \sim B(\overset{n}{17}, \overset{p}{0.2})$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.8^{17}$

$= 0.9775 > 0.9 \quad \therefore$ satisfactory

iv) $X \sim B(19, 0.2)$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.2368$

$= 0.7632 \quad < 0.9$

$X \sim B(18, 0.2)$

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - 0.0991$

$= 0.9009 \quad > 0.9$

Maximum Appointments $= 18$

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v) $H_0 : p = 0.2$

$H_1 : p <> 0.2$

where p is prob a randomly chosen patient fails to turn up

Two tailed test $2\frac{1}{2}\%$ each end

$E(X) = np = 20 \times 0.2 = 4$

$P(X \leq 1) = 0.0692 > 2\frac{1}{2}\%$

Accept H_0

There is insufficient evidence to support the view that the percentage of patients failing to turn up has changed. Accept it is still 20%