Rules of Indices

	Examples
1. $x^p \times x^q = x^{p+q}$	$y^2 \times y^3$ $= (y \times y) \times (y \times y \times y)$ $= y \times y \times y \times y \times y$ $= y^5$
2. $x^p \div x^q = x^{p-q}$	$y^5 \div y^3$ $= \dfrac{y \times y \times y_1 \times y_1 \times y_1}{y_1 \times y_1 \times y_1}$ $= y^2$
3. $(x^p)^q = x^{p \times q}$	$(y^2)^3$ $= y^2 \times y^2 \times y^2$ $= y^6$
4. $x^1 = x$	$y^4 \div y^3 = \dfrac{y \times y \times y \times y}{y \times y \times y}$ $= y$ But by 2nd rule $y^4 \div y^3 = y^{4-3} = y^1$ $\therefore y^1 = y$

5. $x^0 = 1$

$y^3 \div y^3 = 1$

But by 2nd rule

$y^3 \div y^3 = y^{3-3} = y^0$

$\therefore y^0 = 1$

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6. $x^{-p} = \frac{1}{x^p}$

$y^3 \div y^5$

$$\frac{y^1 \times y^1 \times y^1}{y \times y \times y \times y \times y} = \frac{1}{y^2}$$

By second rule

$y^3 \div y^5 = y^{3-5} = y^{-2}$

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7. $x^{\frac{1}{p}} = \sqrt[p]{x}$

$y^{\frac{1}{2}} \times y^{\frac{1}{2}} = y^{\frac{1}{2}+\frac{1}{2}} = y^1 = y$

Also

$y^{\frac{1}{3}} \times y^{\frac{1}{3}} \times y^{\frac{1}{3}} = y^1 = y$

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8. $x^{p/q} = (\sqrt[q]{x})^p$

or $\sqrt[q]{x^p}$

$y^{2/3} = \left(y^{\frac{1}{3}}\right)^2$

$= \left(\sqrt[3]{y}\right)^2$

or $y^{2/3} = \sqrt[3]{y^2}$

Summary

1. $x^p \times x^q = x^{p+q}$
2. $x^p \div x^q = x^{p-q}$
3. $(x^p)^q = x^{p \times q}$
4. $x^1 = x$
5. $x^0 = 1$
6. $x^{-p} = \frac{1}{x^p}$
7. $x^{\frac{1}{p}} = \sqrt[p]{x}$
8. $x^{p/q} = (\sqrt[q]{x})^p$ or $\sqrt[3]{x^p}$

Examples

1) $2x^2 \times 3x^3$

$= 2 \times x^2 \times 3 \times x^3 = 6x^5$

2) $\frac{10x^{10}}{5x^5} = 2x^5$

3) $(2x^2)^3 = 2x^2 \times 2x^2 \times 2x^2$

$= 8x^6$

4) $8^1 = 8$

5) $6^0 = 1$

6) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

7) $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

8) $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

9) $25^{-\frac{3}{2}} = \frac{1}{25^{3/2}} = \frac{1}{(\sqrt{25})^3} = \frac{1}{5^3} = \frac{1}{125}$

10) $\left(\frac{27}{8}\right)^{-\frac{1}{3}} = \left(\frac{8}{27}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

Know your cubes

$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$ $5^3 = 125$ $6^3 = 216$

Blue book P214 Ex 10C

1e) $5^{-2} \times 5^{-3} = 5^{-2+-3} = 5^{-2-3} = 5^{-5}$

2e) $6^{-2} \div 6^{-5} = 6^{-2--5} = 6^{-2+5} = 6^{3}$

3e) $a^{3} \div a = a^{3} - a^{1} = a^{3-1} = a^{2}$

4e) $(4^{-2})^{-3} = 4^{-2 \times -3} = 4^{6}$

5e) $-4a^{3} \times -2a^{5} = 8a^{3+5} = 8a^{8}$

6e) $24a^{5} \div 6a^{-2} = 4a^{5--2} = 4a^{5+2} = 4a^{7}$