Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mathematics

Advanced Subsidiary
Paper 1: Pure Mathematics

Solutions

Specimen Paper
Time: 2 hours

Paper Reference
8MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1. A curve has equation

$$y = 2x^3 - 2x^2 - 2x + 8$$

(a) Find $\dfrac{dy}{dx}$

(2)

(b) Hence find the range of values of x for which y is increasing.
Write your answer in set notation.

(4)

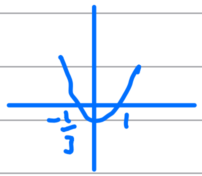
a) $\dfrac{dy}{dx} = 6x^2 - 4x - 2$

b) Increasing function when

$6x^2 - 4x - 2 > 0$

$3x^2 - 2x - 1 > 0$

$(3x + 1)(x - 1) > 0$



$x < -\frac{1}{3}$ or $x > 1$

$\{x : x < -\frac{1}{3}\} \cup \{x : x > 1\}$

2. The quadrilateral $OABC$ has $\overrightarrow{OA} = 4\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{OB} = 6\mathbf{i} - 3\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} - 20\mathbf{j}$.

(a) Find $\overrightarrow{AB}$.

(2)

(b) Show that quadrilateral $OABC$ is a trapezium.

(2)

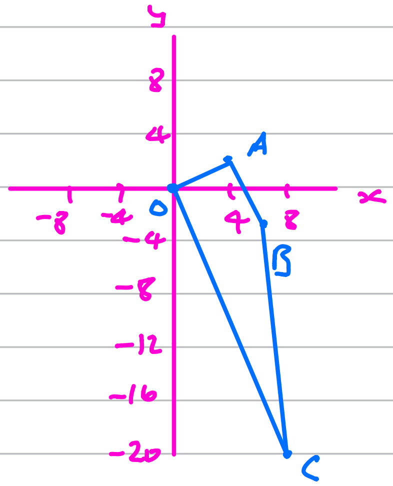
a) $\overrightarrow{OA} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 8 \\ -20 \end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix} -4 \\ -2 \end{pmatrix} + \begin{pmatrix} 6 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$\overrightarrow{AB} = 2\underline{i} - 5\underline{j}$

b)



$\overrightarrow{AB} = 2\underline{i} - 5\underline{j}$

$\overrightarrow{OC} = 8\underline{i} - 20\underline{j}$

$= 4\overrightarrow{AB}$

$\therefore$ $\overrightarrow{OC}$ parallel to $\overrightarrow{AB}$

but not same length

$\therefore$ OABC is a trapezium

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3. A tank, which contained water, started to leak from a hole in its base.

The volume of water in the tank 24 minutes after the leak started was $4\,\text{m}^3$

The volume of water in the tank 60 minutes after the leak started was $2.8\,\text{m}^3$

The volume of water, $V\,\text{m}^3$, in the tank t minutes after the leak started, can be described by a linear model between V and t.

(a) Find an equation linking V with t.

(4)

Use this model to find

(b) (i) the initial volume of water in the tank,

(ii) the time taken for the tank to empty.

(3)

(c) Suggest a reason why this linear model may not be suitable.

(1)

a) $(24, 4)$ $(60, 2.8)$

gradient $\frac{2.8 - 4}{60 - 24} = -\frac{1.2}{36}$

$= -\frac{1}{30}$

$V = -\frac{1}{30}t + c$

Sub $(24,4)$ $4 = -\frac{1}{30} \times 24 + c$

$4\frac{4}{5} = c$

$c = \frac{24}{5}$

$V = -\frac{t}{30} + \frac{24}{5}$

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Question 3 continued

b) i) $t = 0$ $V = 4.8\ m^3$

ii)

$$0 = -\frac{1}{30}t + 4.8$$

$$\frac{t}{30} = 4.8$$

$$t = 144 \text{ min}$$

c) flow could slow as volume decreases because water pressure at site of leak would reduce.

(Total for Question 3 is 8 marks)

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4.

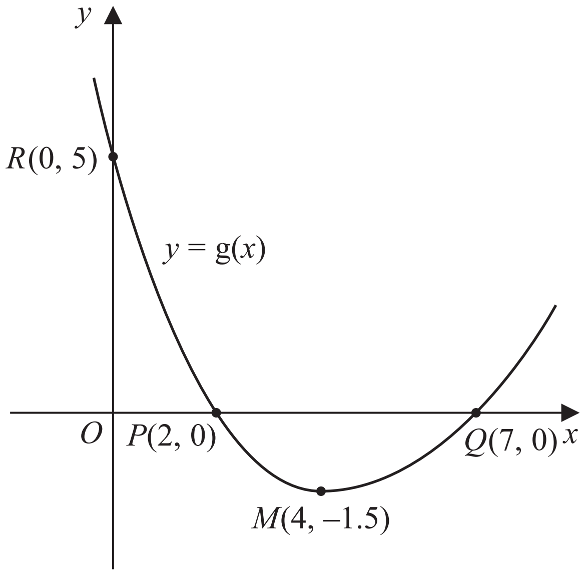

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{g}(x)$.

The curve has a single turning point, a minimum, at the point $M(4, -1.5)$.

The curve crosses the x-axis at two points, $P(2, 0)$ and $Q(7, 0)$.

The curve crosses the y-axis at a single point $R(0, 5)$.

(a) State the coordinates of the turning point on the curve with equation $y = 2\mathrm{g}(x)$.

(1)

(b) State the largest root of the equation

$$\mathrm{g}(x + 1) = 0$$

(1)

(c) State the range of values of x for which $\mathrm{g}'(x) \leqslant 0$

(1)

Given that the equation $\mathrm{g}(x) + k = 0$, where k is a constant, has no real roots,

(d) state the range of possible values for k.

(1)

a) $(4, -3)$

b) $x = 6$

c) $x \leq 4$ d) $k > 1.5$

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5.

$$f(x) = x^3 + 3x^2 - 4x - 12$$

(a) Using the factor theorem, explain why f(x) is divisible by (x + 3).

(2)

(b) Hence fully factorise f(x).

(3)

(c) Show that $\dfrac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x}$ can be written in the form $A + \dfrac{B}{x}$ where A and B are integers to be found.

(3)

a) $f(-3) = (-3)^3 + 3(-3)^2 - 4(-3) - 12$

$= -27 + 27 + 12 - 12$

$= 0$

by factor theorem $(x+3)$ is a factor

$$\begin{array}{r|l} & x^2 \quad - 4 \\ \hline x+3 & x^3 + 3x^2 - 4x - 12 \\ & \underline{x^3 + 3x^2} \\ & \qquad\qquad -4x - 12 \\ & \qquad\qquad \underline{-4x - 12} \end{array}$$

$f(x) = (x+3)(x^2-4)$

$f(x) = (x+3)(x+2)(x-2)$

$$\frac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x} = \frac{(x+3)(x+2)(x-2)}{x(x^2+5x+6)}$$

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Question 5 continued

$$= \frac{(\cancel{x+3})(\cancel{x+2})(x-2)}{x(\cancel{x+2})(\cancel{x+3})}$$

$$= \frac{x-2}{x}$$

$$= 1 - \frac{2}{x}$$

$A = 1$, $B = -2$

(Total for Question 5 is 8 marks)

6. (i) Use a counter example to show that the following statement is false.

"$n^2 - n - 1$ is a prime number, for $3 \leqslant n \leqslant 10$."

(2)

(ii) Prove that the following statement is always true.

"The difference between the cube and the square of an odd number is even."

For example $5^3 - 5^2 = 100$ is even.

(4)

i) $n = 8$ $\quad n^2 - n - 1 = 64 - 8 - 1 = 55$

which is not prime since $11 \times 5 = 55$

ii) Let any odd number be $2n+1$

where n is an integer

$$(2n+1)^3 - (2n+1)^2$$

$$= (2n)^3 + 3(2n)^2 + 3(2n) + 1 - (4n^2 + 4n + 1)$$

$$= 8n^3 + 12n^2 + 6n + 1 - 4n^2 - 4n - 1$$

$$= 8n^3 + 8n^2 + 2n$$

$$= 2(4n^3 + 4n^2 + 1)$$

which is even since 2 is a factor

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7. (a) Expand $\left(1+\frac{3}{x}\right)^2$ simplifying each term.

(2)

(b) Use the binomial expansion to find, in ascending powers of x, the first four terms in the expansion of

$$\left(1+\frac{3}{4}x\right)^6$$

simplifying each term.

(4)

(c) Hence find the coefficient of x in the expansion of

$$\left(1+\frac{3}{x}\right)^2\left(1+\frac{3}{4}x\right)^6$$

(2)

a) $\left(1+\frac{3}{x}\right)^2 = 1 + \frac{6}{x} + \frac{9}{x^2}$

b) $\left(1+\frac{3x}{4}\right)^6$

1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$= 1 + 6\left(\frac{3x}{4}\right) + 15\left(\frac{3x}{4}\right)^2 + 20\left(\frac{3x}{4}\right)^3 + \ldots$

$= 1 + \frac{9x}{2} + \frac{135x^2}{16} + \frac{135}{16}x^3 + \ldots$

c) $\left(1+\frac{6}{x}+\frac{9}{x^2}\right)\left(1+\frac{9}{2}x+\frac{135x^2}{16}+\frac{135x^3}{16}+\ldots\right)$

Coeff of x $= \frac{9}{2} + \frac{405}{8} + \frac{1215}{16} = \frac{2097}{16}$

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8.

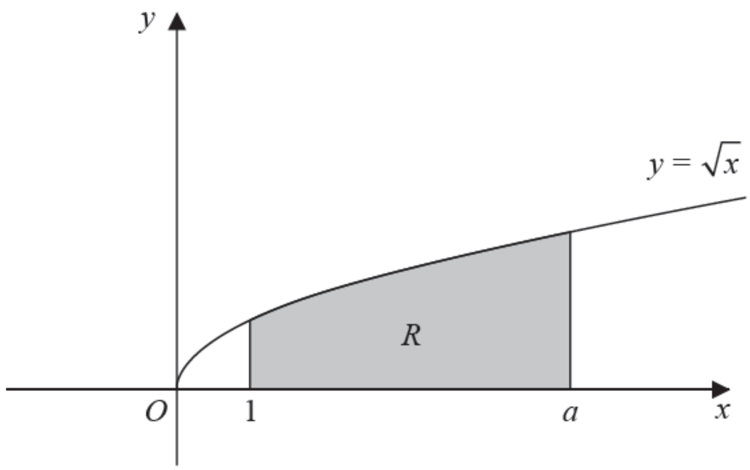

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{x}$, $x \geqslant 0$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = a$, where a is a constant.

Given that the area of R is 10

(a) find, in simplest form, the value of

(i) $\int_1^a \sqrt{8x}\, dx$

(ii) $\int_0^a \sqrt{x}\, dx$

(4)

(b) show that $a = 2^k$, where k is a rational constant to be found.

(4)

i) $\int_1^a \sqrt{8x}\, dx = \sqrt{8}\int_1^a \sqrt{x}\, dx = 10\sqrt{8}$

$= 20\sqrt{2}$

ii) $\int_0^a \sqrt{x}\, dx = \int_0^1 \sqrt{x}\, dx + \int_1^a \sqrt{x}\, dx$

$= \int_0^1 x^{\frac{1}{2}}\, dx + 10$

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Question 8 continued

$$= \left[\frac{x^{3/2}}{3/2}\right]_0^1 + 10$$

$$= \left[\frac{2}{3}x^{3/2}\right]_0^1 + 10$$

$$= \left(\frac{2}{3} - 0\right) + 10$$

$$= \frac{32}{3}$$

b) $$\int_1^a \sqrt{x}\,dx = 10$$

$$\left[\frac{2}{3}x^{3/2}\right]_1^a = 10$$

$$\frac{2}{3}a^{3/2} - \frac{2}{3} = 10$$

$$\frac{2}{3}a^{3/2} = \frac{32}{3}$$

$$a^{3/2} = 16$$

$$a = 16^{2/3} = \left(2^4\right)^{2/3} = 2^{8/3}$$

$$k = \frac{8}{3}$$

(Total for Question 8 is 8 marks)

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9. Find any real values of x such that

$$2\log_4(2-x) - \log_4(x+5) = 1$$

(6)

$$\log_4 (2-x)^2 - \log_4 (x+5) = 1$$

$$\log_4 \left(\frac{(2-x)^2}{(x+5)} \right) = 1$$

$$\frac{(2-x)^2}{x+5} = 4^1 = 4$$

$$4 - 4x + x^2 = 4(x+5)$$

$$4 - 4x + x^2 = 4x + 20$$

$$x^2 - 8x - 16 = 0$$

By calc

$$x = 4 \pm 4\sqrt{2}$$

$x \approx 9.66$ or $x = -1.66$

$\ln(2-x)$ not defined for $x = 9.66$

$\therefore$ only solution $\underline{x = 4 - 4\sqrt{2}}$

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10. A circle C has centre (2, 5). Given that the point $P(-2, 3)$ lies on C.

(a) find an equation for C.

(3)

The line l is the tangent to C at the point P. The point $Q(2, k)$ lies on l.

(b) Find the value of k.

(5)

a) Radius = distance (2, 5) to (−2, 3)

$= \sqrt{(2--2)^2+(5-3)^2} = \sqrt{16+4} = \sqrt{20}$

Circle C $\quad (x-2)^2 + (y-5)^2 = 20$

b) Gradient of radius to P

$= \dfrac{5-3}{2--2} = \dfrac{2}{4} = \dfrac{1}{2}$

Gradient of tangent ∴ is -2

$y - y_1 = m(x - x_1)$

$P(-2,3)$ $\quad y - 3 = -2(x--2)$

$y - 3 = -2(x+2)$

$y - 3 = -2x - 4$

tangent l $\quad y = -2x - 1$

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Question 10 continued

$Q(2, k)$ on tangent l

$k = -2(2) - 1$

$k = -4 - 1$

$k = -5$

(Total for Question 10 is 8 marks)

11. (i) Solve, for $-90° \leqslant \theta < 270°$, the equation,

$$\sin(2\theta + 10°) = -0.6$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt at the question

"Solve, for $-90° < x < 90°$, the equation $7\tan x = 8\sin x$"

is set out below.

$$7\tan x = 8\sin x$$

$$7 \times \frac{\sin x}{\cos x} = 8\sin x$$

$$7\sin x = 8\sin x\cos x$$

$$7 = 8\cos x$$

$$\cos x = \frac{7}{8}$$

$$x = 29.0° \text{ (to 3 sf)}$$

Identify two mistakes made by this student, giving a brief explanation of each mistake.

(2)

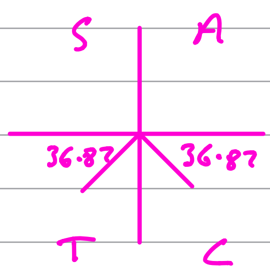
(b) Find the smallest positive solution to the equation

$$7\tan(4\alpha + 199°) = 8\sin(4\alpha + 199°)$$

(2)

i) $\sin(2\theta + 10°) = -0.6$

$\sin^{-1}(0.6) = 36.87°$



$2\theta + 10° = -143.13, \ -36.87°, \ 216.87°, \ 323.13°$

$2\theta = -153.13° \ -46.87°, \ 206.87°, \ 313.13°$

Question 11 continued

$\theta = -76.6^\circ, -23.4^\circ, 103.4^\circ, 156.6^\circ$

ii) a) Divided by variable term sinx instead of factorising, thereby failing to find root when $\sin x = 0, \Rightarrow x = 0$

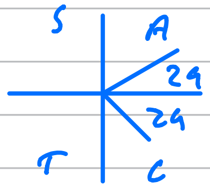
Failed to note $\cos(-x) = \cos x$

so $x = -29.0^\circ$ also a root

b)

$$\cos(4\alpha + 199^\circ) = \frac{7}{8}$$

$\cos^{-1}\frac{7}{8} = 29^\circ$



$29^\circ, 331^\circ$

$-29^\circ, -331^\circ$

$4\alpha + 199 = 331$

$4\alpha = 331 - 199$

$4\alpha = 132$

$\alpha = 33^\circ$

(Total for Question 11 is 9 marks)

12.

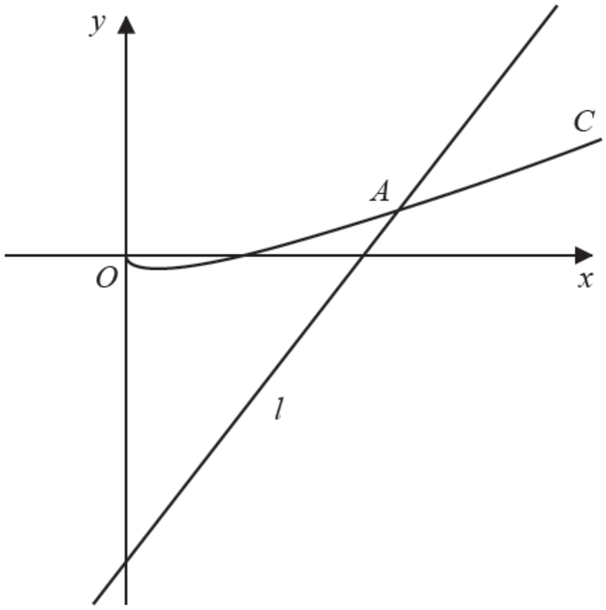


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = 3x - 2\sqrt{x}$, $x \geqslant 0$ and the line l with equation $y = 8x - 16$

The line cuts the curve at point A as shown in Figure 3.

(a) Using algebra, find the x coordinate of point A.

(5)

(b)

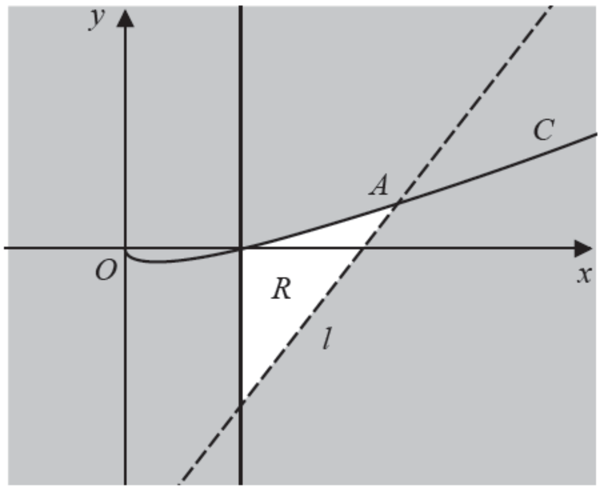


Figure 4

The region R is shown unshaded in Figure 4. Identify the inequalities that define R.

(3)

a) $y = 3x - 2\sqrt{x}$ ①

$y = 8x - 16$ ②

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Question 12 continued

sub for y in ①

$8x - 16 = 3x - 2\sqrt{x}$

$5x + 2\sqrt{x} - 16 = 0$

Quadratic in $\sqrt{x}$

By calc $\sqrt{x} = \frac{8}{5}$ or ~~$\sqrt{x} = -2$~~

$\Rightarrow x = \left(\frac{8}{5}\right)^2 = \frac{64}{25}$

x coord of $A = \frac{64}{25}$

b) $y = 3x - 2\sqrt{x}$

$y = 0 \Rightarrow 3x - 2\sqrt{x} = 0$

$\sqrt{x}(3\sqrt{x} - 2) = 0$

$\sqrt{x} = 0 \Rightarrow x = 0$

or $3\sqrt{x} - 2 = 0$

$3\sqrt{x} = 2 \Rightarrow \sqrt{x} = \frac{2}{3} \Rightarrow x = \frac{4}{9}$

Inequalities:

$x \geqslant \frac{4}{9}$

$y > 8x - 16$

$y \leq 3x - 2\sqrt{x}$

Question 12 continued

(Total for Question 12 is 8 marks)

13. The growth of pond weed on the surface of a pond is being investigated.

The surface area of the pond covered by the weed, A m², can be modelled by the equation

$$A = 0.2e^{0.3t}$$

where t is the number of days after the start of the investigation.

(a) State the surface area of the pond covered by the weed at the start of the investigation.
(1)

(b) Find the rate of increase of the surface area of the pond covered by the weed, in m²/day, exactly 5 days after the start of the investigation.
(2)

Given that the pond has a surface area of 100 m²,

(c) find, to the nearest hour, the time taken, according to the model, for the surface of the pond to be fully covered by the weed.
(4)

The pond is observed for one month and by the end of the month 90% of the surface area of the pond was covered by the weed.

(d) Evaluate the model in light of this information, giving a reason for your answer.
(1)

a) $t=0$, $A = 0.2e^{0} = 0.2 \text{ m}^2$

b) $A = 0.2e^{0.3t}$

$\frac{dA}{dt} = 0.06e^{0.3t}$

$t=5$, $\frac{dA}{dt} = 0.06e^{1.5} = 0.269 \text{ m}^2/\text{day}$

c) $100 = 0.2e^{0.3t}$

$500 = e^{0.3t}$

$\ln 500 = 0.3t$

Question 13 continued

$$t = \frac{\ln 500}{0.3}$$

$$t = 20.715 \text{ days}$$

$$t = 20 \text{ days } 17 \text{ hours}$$

d) Model unsuitable

Predicts 100% within 21 days yet actually only 90% in 30 days

(Total for Question 13 is 8 marks)

14.

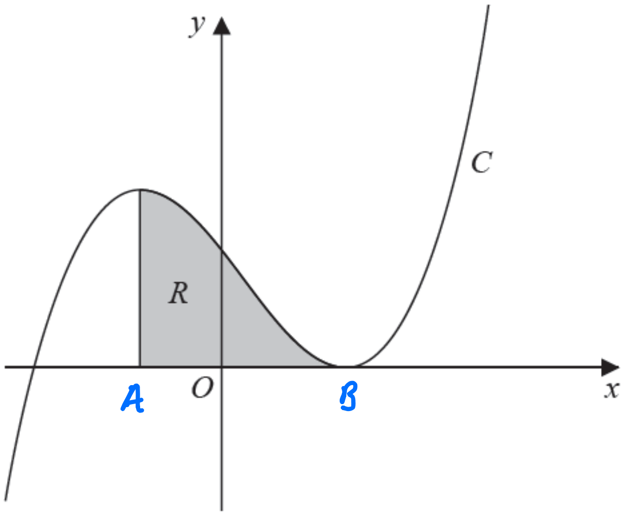

Figure 5

Figure 5 shows a sketch of the curve C with equation $y = (x - 2)^2(x + 3)$

The region R, shown shaded in Figure 5, is bounded by C, the vertical line passing through the maximum turning point of C and the x-axis.

Find the exact area of R.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(9)

$B(2, 0)$

$y = (x^2 - 4x + 4)(x + 3)$

$y = x^3 - 4x^2 + 4x$
$\quad + 3x^2 - 12x + 12$

$y = x^3 - x^2 - 8x + 12$

$\frac{dy}{dx} = 3x^2 - 2x - 8$

At t.p. $\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 2x - 8 = 0$

by calc $x = 2$ or $x = -\frac{4}{3}$

$\therefore A(-\frac{4}{3}, 0)$

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Question 14 continued

$$R = \int_{-\frac{4}{3}}^{2} y \, dx$$

$$= \int_{-\frac{4}{3}}^{2} \left(x^3 - x^2 - 8x + 12\right) dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - 4x^2 + 12x \right]_{-\frac{4}{3}}^{2}$$

$$= \left(\frac{16}{4} - \frac{8}{3} - 16 + 24 \right) - \left(\frac{\left(-\frac{4}{3}\right)^4}{4} - \frac{\left(-\frac{4}{3}\right)^3}{3} - 4\left(-\frac{4}{3}\right)^2 + 12\left(-\frac{4}{3}\right) \right)$$

$$= \frac{28}{3} - \left(\frac{64}{81} + \frac{64}{81} - \frac{64}{9} - \frac{48}{3} \right)$$

$$= \frac{28}{3} + \frac{1744}{81}$$

$$= \frac{2500}{81}$$

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