

1. Prove that  $(n + 4)^2 - (3n + 4) = (n + 1)(n + 4) + 8$
2. Prove that  $(n + 4)^2 - (3n + 4) = (n + 2)(n + 3) + 6$
3. Prove that  $(n + 3)^2 - (3n + 5) = (n + 1)(n + 2) + 2$
4. Prove that  $(n - 5)^2 - (2n - 1) = (n - 3)(n - 9) - 1$
5. Prove that  $(n - 3)^2 - (n - 5) = (n - 3)(n - 4) + 2$
6. Prove that  $\frac{1}{2}(n + 1)(n + 2) - \frac{1}{2}n(n + 1) = n + 1$
7. Prove that  $\frac{1}{4}(2n + 1)(n + 4) - \frac{1}{4}n(2n + 1) = 2n + 1$
  
8. Prove that  $(3n + 1)^2 - (3n - 1)^2$  is a multiple of 6 for all positive integer values of  $n$ .
9. Prove that  $(4n + 1)^2 - (4n - 1)^2$  is a multiple of 8 for all positive integer values of  $n$ .
10. Prove that  $(5n + 1)^2 - (5n - 1)^2$  is a multiple of 5 for all positive integer values of  $n$ .

1. Prove that  $(n + 4)^2 - (3n + 4) = (n + 1)(n + 4) + 8$

$$\begin{aligned}(n+4)^2 - (3n+4) &= n^2 + 8n + 16 - 3n - 4 \\&= \underline{n^2 + 5n + 12} \quad (*)\end{aligned}$$

$$\begin{aligned}(n+1)(n+4) + 8 &= n^2 + n + 4n + 4 + 8 \\&= \underline{n^2 + 5n + 12} \quad (**)\end{aligned}$$

Proved since  $(*) = (**)$

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2. Prove that  $(n + 4)^2 - (3n + 4) = (n + 2)(n + 3) + 6$

$$\begin{aligned}(n+4)^2 - (3n+4) &= n^2 + 8n + 16 - 3n - 4 \\&= \underline{n^2 + 5n + 12} \quad *\end{aligned}$$

$$\begin{aligned}(n+2)(n+3) + 6 &= n^2 + 2n + 3n + 6 + 6 \\&= n^2 + 5n + 12 \quad **\end{aligned}$$

Proved since  $* = **$

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3. Prove that  $(n + 3)^2 - (3n + 5) = (n + 1)(n + 2) + 2$

$$\begin{aligned}(n+3)^2 - (3n+5) &= n^2 + 6n + 9 - 3n - 5 \\&= n^2 + 3n + 4 \quad *\end{aligned}$$

$$\begin{aligned}(n+1)(n+2) + 2 &= n^2 + n + 2n + 2 + 2 \\&= n^2 + 3n + 4 \quad **\end{aligned}$$

Proved since  $* = **$

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4. Prove that  $(n - 5)^2 - (2n - 1) = (n - 3)(n - 9) - 1$

$$\begin{aligned}(n - 5)^2 - (2n - 1) &= n^2 - 10n + 25 - 2n + 1 \\&= \underline{n^2 - 12n + 26} \quad *\end{aligned}$$

$$\begin{aligned}(n - 3)(n - 9) - 1 &= n^2 - 3n - 9n + 27 - 1 \\&= \underline{n^2 - 12n + 26} \quad **\end{aligned}$$

Proved since  $* = **$

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5. Prove that  $(n - 3)^2 - (n - 5) = (n - 3)(n - 4) + 2$

$$\begin{aligned}(n - 3)^2 - (n - 5) &= n^2 - 6n + 9 - n + 5 \\&= \underline{n^2 - 7n + 14} \quad *\end{aligned}$$

$$\begin{aligned}(n - 3)(n - 4) + 2 &= n^2 - 3n - 4n + 12 + 2 \\&= \underline{n^2 - 7n + 14} \quad **\end{aligned}$$

Proved since  $* = **$

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6. Prove that  $\frac{1}{2}(n + 1)(n + 2) - \frac{1}{2}n(n + 1) = n + 1$

$$\begin{aligned}&\frac{1}{2}(n^2 + n + 2n + 2) - \frac{1}{2}(n^2 + n) \\&= \frac{1}{2}(n^2 + 3n + 2) - \frac{1}{2}(n^2 + n) \\&= \cancel{\frac{1}{2}n^2} + \frac{3n}{2} + 1 - \cancel{\frac{1}{2}n^2} - \frac{1}{2}n = n + 1\end{aligned}$$

7. Prove that  $\frac{1}{4}(2n+1)(n+4) - \frac{1}{4}n(2n+1) = 2n + 1$

$$\begin{aligned}
 & \frac{1}{4}(2n^2 + n + 8n + 4) - \frac{1}{4}(2n^2 + n) \\
 &= \frac{1}{4}(2n^2 + 9n + 4) - \frac{1}{4}(2n^2 + n) \\
 &= \cancel{\frac{1}{2}n^2} + \frac{9}{4}n + 1 - \cancel{\frac{1}{2}n^2} - \frac{1}{4}n \\
 &= 2n + 1
 \end{aligned}$$


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8. Prove that  $(3n+1)^2 - (3n-1)^2$  is a multiple of 6 for all positive integer values of  $n$ .

$$\begin{aligned}
 & (3n+1)^2 - (3n-1)^2 \\
 &= (9n^2 + 6n + 1) - (9n^2 - 6n + 1) \\
 &= \cancel{9n^2} + 6n + 1 - \cancel{9n^2} + 6n - 1 \\
 &= 12n \\
 &= 6(2n) \quad \therefore \text{ a multiple of 6}
 \end{aligned}$$


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9. Prove that  $(4n+1)^2 - (4n-1)^2$  is a multiple of 8 for all positive integer values of  $n$ .

$$\begin{aligned}
 & (4n+1)^2 - (4n-1)^2 \\
 &= (16n^2 + 8n + 1) - (16n^2 - 8n + 1) \\
 &= \cancel{16n^2} + 8n + 1 - \cancel{16n^2} + 8n - 1
 \end{aligned}$$

$$= 16n$$

$$= 8(2n) \quad \therefore \text{ a multiple of } 8$$

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10. Prove that  $(5n + 1)^2 - (5n - 1)^2$  is a multiple of 5 for all positive integer values of  $n$ .

$$(5n+1)^2 - (5n-1)^2$$

$$= (25n^2 + 10n + 1) - (25n^2 - 10n + 1)$$

$$= \cancel{25n^2 + 10n + 1} - \cancel{25n^2 - 10n + 1}$$

$$= 20n$$

$$= 5(4n) \quad \therefore \text{ a multiple of } 5$$

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