Trigonometric Identities

Definitions

secant θ written $\sec\theta = \dfrac{1}{\cos\theta}$

cosecant θ written $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

cotangent θ written $\cot\theta = \dfrac{1}{\tan\theta}$

$$\sin^2\theta + \cos^2\theta = 1$$

$\div \cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \quad \tan^2\theta + 1 = \sec^2\theta$$

$$\underline{1 + \tan^2\theta = \sec^2\theta}$$

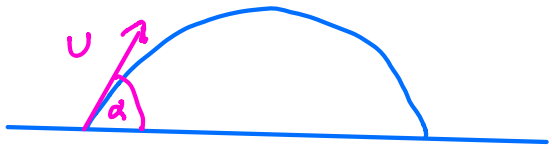
$$\sin^2\theta + \cos^2\theta = 1$$

$\div \sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\Rightarrow \quad \underline{1 + \cot^2\theta = \operatorname{cosec}^2\theta}$$

Cartesian Equation of a Projectile



$$\left(y = ut + \tfrac{1}{2}at^2\right)$$

$$y = U\sin\alpha \times t - 4.9t^2 \quad (1)$$

$$x = U\cos\alpha \times t \quad (2)$$

From (2)

$$t = \frac{x}{U\cos\alpha}$$

Sub for t in (1)

$$y = U\sin\alpha \times \frac{x}{U\cos\alpha} - 4.9\frac{x^2}{U^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{4.9x^2}{U^2}\sec^2\alpha$$

$$y = x\tan\alpha - \frac{4.9x^2}{U^2}\left(1 + \tan^2\alpha\right)$$

Example $U = 10\ ms^{-1}$, $\alpha = 45°$

$$y = x\tan 45° - \frac{4.9x^2}{10^2}\left(1 + \tan^2 45°\right)$$

$$y = x - 0.049x^2(2)$$

$$y = x - 0.098x^2$$

Find horizontal range

Lands when $y = 0$

$$0 = x - 0.098x^2$$

$$0 = x(1 - 0.098x)$$

$\cancel{x = 0}$ or $1 - 0.098x = 0$

$1 = 0.098x$

$x = \frac{1}{0.098}$

$x \quad 10.2 \text{ m}$

More Worked Examples

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5 You should neglect air resistance in this question.

A small stone is projected from ground level. The maximum height of the stone above horizontal ground is 22.5 m.

(i) Show that the vertical component of the initial velocity of the stone is 21 m s^{-1}. [2]

The speed of projection is 28 m s^{-1}.

(ii) Find the angle of projection of the stone. [2]

(iii) Find the horizontal range of the stone. [4]

i)

$(v^2 = u^2 + 2as)$

$v_y^2 = u_y^2 - 19.6y$

At max height $v_y = 0$

$0 = u_y^2 - 19.6 \times 22.5$

$u_y = \sqrt{19.6 \times 22.5}$

$u_y = 21 \text{ ms}^{-1}$

ii) $U_y = U\sin\alpha = 21 \text{ ms}^{-1}$

$\Rightarrow 28\sin\alpha = 21$

$\sin\alpha = \frac{21}{28}$

$\alpha = \sin^{-1}\left(\frac{21}{28}\right)$

$\alpha = 48.6^\circ$

iii) Time of flight calculation

$y = U\sin\alpha\, t - 4.9t^2$

$y = 21t - 4.9t^2$

Lands when $y = 0$

$0 = 21t - 4.9t^2$

$0 = t(21 - 4.9t)$

$\cancel{t = 0}$ or $21 - 4.9t = 0$

$t = \frac{21}{4.9} = \frac{30}{7}$ s

$x = U\cos\alpha \times t$

$x = 28\cos 48.6^\circ \times \frac{30}{7}$

$x = 79.4$ m