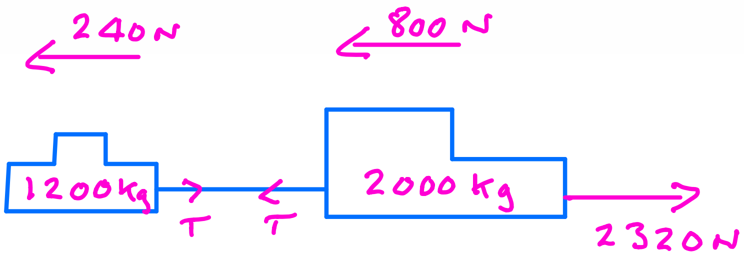
Connected Particles Homework 2 Solutions

1 A breakdown van of mass 2000 kg is towing a car of mass 1200 kg along a straight horizontal road. The two vehicles are joined by a tow bar which remains parallel to the road. The van and the car experience constant resistances to motion of magnitudes 800 N and 240 N respectively. There is a constant driving force acting on the van of 2320 N. Find

(*a*) the magnitude of the acceleration of the van and the car, **(3)**

(*b*) the tension in the tow bar. **(4)**



a) N2L Whole System $F = ma$

$$2320 - 800 - 240 = (1200 + 2000)a$$

$$1280 = 3200a$$

$$\frac{1280}{3200} = a$$

$$\underline{a = 0.4 \text{ ms}^{-2}}$$

b) N2L for car

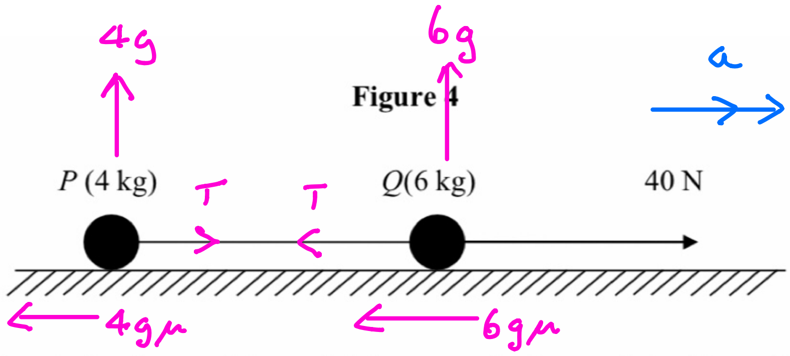
$$T - 240 = 1200 \times 0.4$$

$$T = 480 + 240$$

$$T = 720 \text{ N}$$

2.

Figure 4



Two particles P and Q, of mass 4 kg and 6 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. The coefficient of friction between each particle and the plane is $\frac{2}{7}$. A constant force of magnitude 40 N is then applied to Q in the direction PQ, as shown in Fig. 4.

(*a*) Show that the acceleration of Q is 1.2 m s^{-2}.
(4)

(*b*) Calculate the tension in the string when the system is moving.
(3)

(*c*) State how you have used the information that the string is inextensible.
(1)

After the particles have been moving for 7 s, the string breaks. The particle Q remains under the action of the force of magnitude 40 N.

(*d*) Show that P continues to move for a further 3 seconds.
(5)

(*e*) Calculate the speed of Q at the instant when P comes to rest.
(4)

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a) N2L for whole system $F = ma$

$$40 - 4g\mu - 6g\mu = (4+6)a$$

$$40 - 4\times 9.8\times \frac{2}{7} - 6\times 9.8\times \frac{2}{7} = 10a$$

$$12 = 10a$$

$$\frac{12}{10} = a$$

$$\underline{a = 1.2\text{ ms}^{-2}}$$

b) N2L for P

$$T - 4g\mu = 4\times 1.2$$

$$T = 4\times 1.2 + 4\times 9.8\times \frac{2}{7}$$

$$\underline{T = 16\text{ N}}$$

c) P and Q have same acceleration

d) When $t = 7s$

$$v = u + at$$
$$v = 0 + 1.2 \times 7$$
$$v = 8.4\ ms^{-1}$$

So when string breaks, P and Q both have speed $8.4\ ms^{-1}$

N2L for P

$$-4 \times 9.8 \times \frac{2}{7} = 4a$$

$$-\frac{11.2}{4} = a$$

$$a = -2.8\ ms^{-2}$$

Restart time $t = 0s$, $u = 8.4\ ms^{-1}$, $a = -2.8\ ms^{-2}$

P stops when $v = 0$

$$v = u + at$$
$$0 = 8.4 - 2.8t$$
$$2.8t = 8.4$$
$$t = \frac{8.4}{2.8}$$
$$t = 3s$$

So P moves for further 3 seconds

e) N2L for Q

$$40 - 6 \times 9.8 \times \frac{2}{7} = 6a$$

$$\frac{23.2}{6} = a$$

$$a = \frac{58}{15} \text{ ms}^{-2}$$

$U = 8.4 \text{ ms}^{-1}, \quad a = \frac{58}{15} \text{ ms}^{-2}, \quad t = 3\text{s}$

$$V = U + at$$
$$V = 8.4 + \frac{58}{15} \times 3$$
$$V = 20 \text{ ms}^{-1}$$

Q has speed 20 ms^{-1}

3.

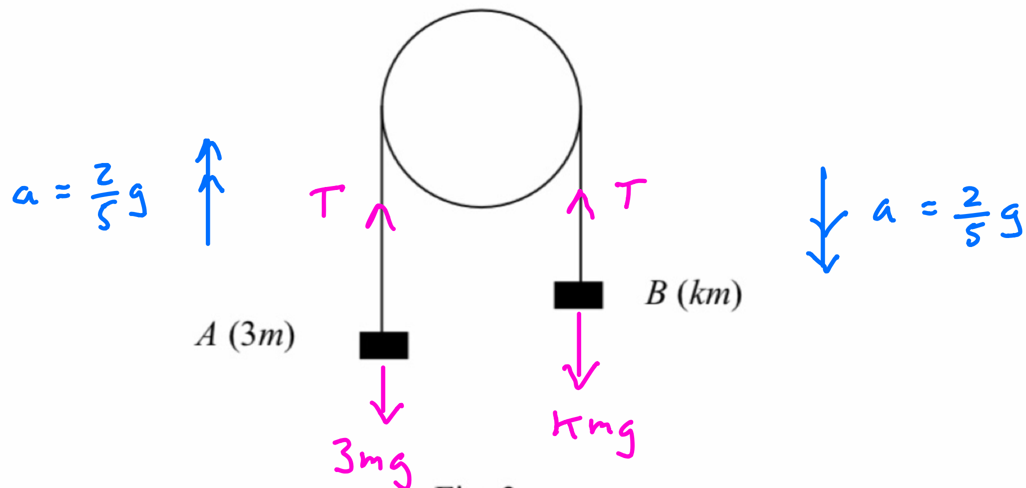


Fig. 3

Two particles A and B have masses $3m$ and km respectively, where $k > 3$. They are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with the string taut and the hanging parts of the string vertical, as shown in Fig. 3. While the particles are moving freely, A has an acceleration of magnitude $\frac{2}{5}g$.

(*a*) Find, in terms of m and g, the tension in the string. **(3 marks)**

(*b*) State why B also has an acceleration of magnitude $\frac{2}{5}g$. **(1 mark)**

(*c*) Find the value of k. **(4 marks)**

(*d*) State how you have used the fact that the string is light. **(1 mark)**

a) N2L for A

$$T - 3mg = 3m \times \frac{2}{5}g$$

$$T = \frac{6}{5}mg + 3mg$$

$$T = \frac{21mg}{5} \text{ N}$$

b) String is inextensible

c) N2L for B

$$kmg - T = km \times \frac{2}{5}g$$

$$kmg - \frac{2}{5}kmg = T$$

$$\frac{3}{5}kmg = \frac{21mg}{5}$$

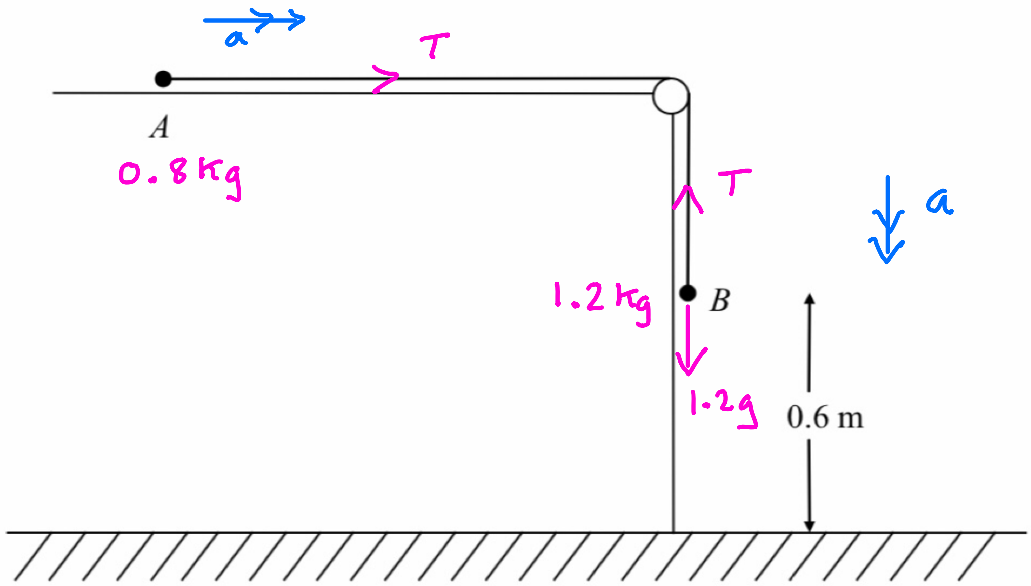
$$k = \frac{21mg}{5} \times \frac{5}{3mg}$$

$$k = 7$$

d) Neglected mass of string and considered only the masses of A and B

4.

Figure 4



A particle A of mass 0.8 kg rests on a horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a particle B of mass 1.2 kg which hangs freely below the pulley, as shown in Fig. 4. The system is released from rest with the string taut and with B at a height of 0.6 m above the ground. In the subsequent motion A does not reach P before B reaches the ground. In an initial model of the situation, the table is assumed to be smooth. Using this model, find

(*a*) the tension in the string before B reaches the ground,

(5)

(*b*) the time taken by B to reach the ground.

(3)

a) For A, $T = 0.8a$

For B, $1.2g - T = 1.2a$

Adding $1.2g = 2a$

$0.6g = a$

$\Rightarrow T = 0.8 \times 0.6g$

$T = 0.48g$

$T = 4.704$

$\underline{T = 4.70 \text{ N}}$

b) For B

$$s = ut + \frac{1}{2}at^2$$

$$0.6 = 0 + \frac{1}{2} \times 0.6 \times 9.8\,t^2$$

$$\frac{1.2}{0.6 \times 9.8} = t^2$$

$$t^2 = \frac{10}{49}$$

$$\underline{t = 0.452\text{ s}}$$

5. A car which has run out of petrol is being towed by a breakdown truck along a straight horizontal road. The truck has mass 1200 kg and the car has mass 800 kg. The truck is connected to the car by a horizontal rope which is modelled as light and inextensible. The truck's engine provides a constant driving force of 2400 N. The resistances to motion of the truck and the car are modelled as constant and of magnitude 600 N and 400 N respectively. Find

(*a*) the acceleration of the truck and the ear,

(3)

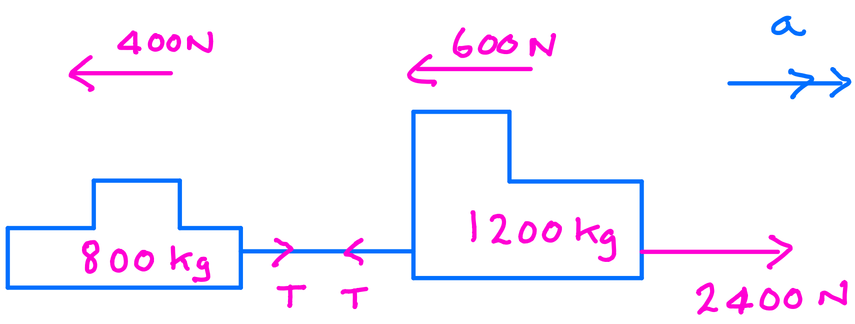
(*b*) the tension in the rope.

(3)

When the truck and car are moving at 20 m s^{-1}, the rope breaks. The engine of the truck provides the same driving force as before. The magnitude of the resistance to the motion of the truck remains 600 N.

(*c*) Show that the truck reaches a speed of 28 m s^{-1} approximately 6 s earlier than it would have done if the rope had not broken.

(7)



a) N2L for whole system

$$2400 - 600 - 400 = (800 + 1200)a$$

$$1400 = 2000a$$

$$a = \frac{1400}{2000}$$

$$\underline{a = 0.7 \text{ ms}^{-2}}$$

b) N2L for car

$$T - 400 = 800 \times 0.7$$

$$T = 800 \times 0.7 + 400$$

$$\underline{T = 960 \text{ N}}$$

c) Start clock when rope breaks

$t = 0\text{s}$, $u = 20 \text{ ms}^{-1}$,

If rope does not snap $a_1 = 0.7 \text{ ms}^{-2}$

$$v = u + at$$

$$28 = 20 + 0.7t$$

$$\frac{8}{0.7} = t$$

$$\underline{t = \frac{80}{7} = 11\tfrac{3}{7} \text{ s}}$$

If rope snaps, N2L for truck

$$2400 - 600 = 1200a$$

$$\frac{1800}{1200} = a$$

$$a = 1.5 \text{ ms}^{-2}$$

$$V = u + at$$

$$28 = 20 + 1.5t$$

$$\frac{8}{1.5} = t$$

$$t = \frac{80}{15} = \frac{16}{3} = 5\tfrac{1}{3}\ s$$

$$\text{Time difference} = 11\tfrac{3}{7} - 5\tfrac{1}{3} = 6.095\ s$$

$$\approx 6\ s$$

So if rope snaps, truck reaches speed of $28\ ms^{-1}$ approximately 6 seconds quicker than if rope does not snap.
